Active & Passive element :-

- An active element is one which is capable of delivering net amount of power to the external device.

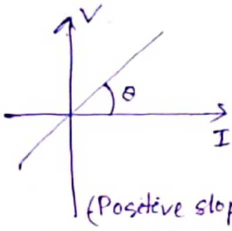
  Ex: voltage source, current source, battery etc.

- Passive elements are those which are capable ~~of~~ only of receiving power.

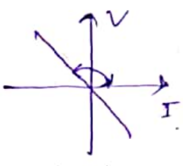
  Ex: R, L, C only if $R \geq 0$, $L \geq 0$, $C \geq 0$.

* The V-I characteristics graph of a passive ~~an active~~ element has positive slope i.e. graph will lie in 1st and/or 3rd quadrant.

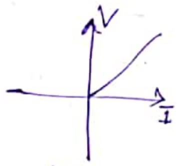
- The V-I characteristics graph of an active element has negative slope i.e. graph will lie in other than 1st & 3rd quadrant.



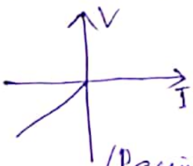
(Positive slope)
(Passive element)



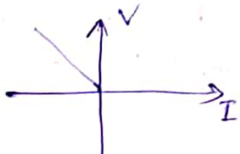
(Active)



(Passive)



(Passive)



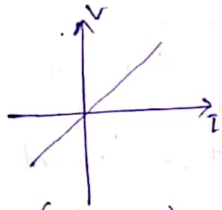
(Active)

Linear & Non Linear :-

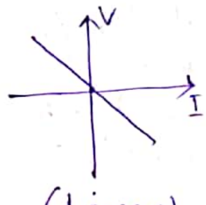
- An element is said to be Linear if it's voltage-current (V-I) characteristics follows only one equation of straight line passing through origin for all the time.

- A Linear element or network satisfies the principle of superposition i.e. principle of ~~homogeneity~~ homogeneity and additivity.

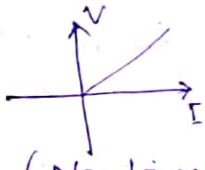
- The element which doesn't satisfy above conditions is said to be non-Linear.



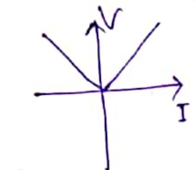
(Linear)



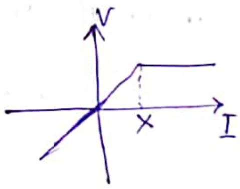
(Linear)



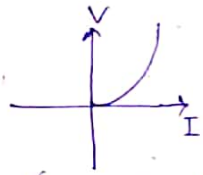
(Non-Linear as for $V<0$, $I=0$)



(Non-Linear as it is having 2 equations)

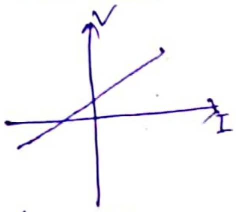


(Non-Linear)
After $x>0$, it is not following equation of straight Line, it is constant.



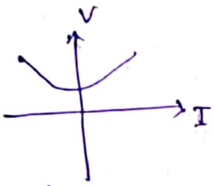
(Non-Linear)
(Not the equation of straight Line)

~~Active element & Passive~~



(Non-Linear)

⇒ Equation doesn't pass through origin.



(Non-Linear)

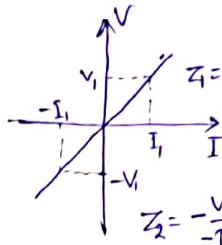
BiLateral & UniLateral :-

- In the bilateral element, the voltage-current relation is the same for current flowing in either direction i.e. it offers same impedance when direction/polarity of voltage/current gets reversed.

- In contrast, a unilateral element has different relations between voltage and current for two possible direction of current.

ex:- Bilateral :- R, L, C.

Unilateral : Diode.

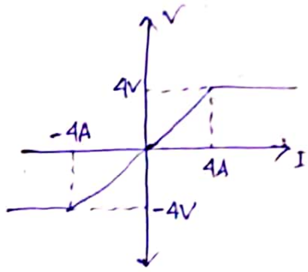


$Z_1 = \frac{V_1}{I_1}$

$Z_2 = \frac{-V_1}{-I_1} = \frac{V_1}{I_1}$

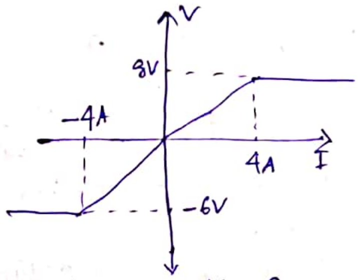
$\Rightarrow Z_1 = Z_2$

(Bilateral)



$\frac{V}{I} = \frac{4}{4} = 1\Omega$ for all

(Bilateral)

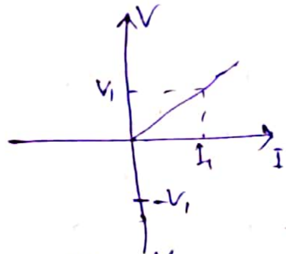


At $i = 4A$, $Z_1 = \frac{V}{I} = \frac{8}{4} = 2\Omega$

When current is reversed i.e.

at $i = -4A$, $Z_2 = \frac{V}{I} = \frac{-6}{-4} = 1.5\Omega$

$Z_1 \neq Z_2 \Rightarrow$ Unilateral element.



$Z_1 = \frac{V_1}{I_1}$

At $-V_1 \Rightarrow -I_1 = 0$

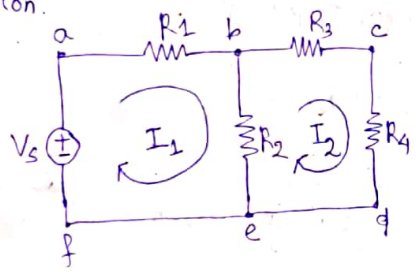
$\Rightarrow Z_2 = \frac{V}{I} = \infty\Omega$

$Z_1 \neq Z_2 \Rightarrow$ Unilateral element)

Note:- Every Linear element is bilateral element but converse is not true.

Mesh Analysis :-

- Mesh and nodal analysis are two basic important techniques used in finding solutions for a network.
- It is useful to use mesh analysis when a network has a large number of voltage sources. On the other hand, if the network has more current sources, nodal analysis is more useful.
- To apply mesh analysis, select the no. of meshes present and assign mesh currents to each of the meshes. Then, writing Kirchhoff's voltage law equations in terms of unknown currents for each mesh and solving them leads to the final solution.



In the above figure, there are two loops i.e. abefa and bcdeb. Assuming loop currents I_1 and I_2 with direction as indicated.

In the Loop abefa,

Current I_1 is passing through R_1 and $(I_1 - I_2)$ is passing through R_2. Applying KVL in the loop,

$$-V_s + I_1R_1 + (I_1 - I_2)R_2 = 0$$

$$\Rightarrow I_1R_1 + (I_1 - I_2)R_2 = V_s$$

$$\Rightarrow (R_1 + R_2)I_1 - I_2R_2 = V_s \quad \text{——— (1)}$$

In the Loop bcdeb;

Current I_2 is passing through R_3 and R_4, and $(I_2 - I_1)$ is passing through R_2. Applying KVL,

$$(I_2 - I_1)R_2 + I_2R_3 + I_2R_4 = 0$$

$$\Rightarrow -R_2I_1 + (R_2 + R_3 + R_4)I_2 = 0 \quad \text{——— (2)}$$

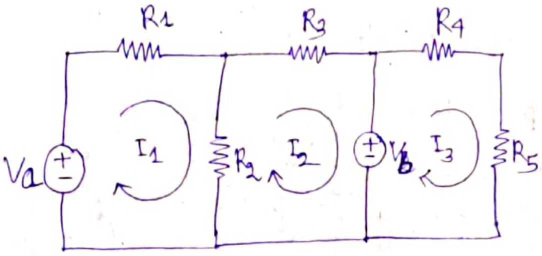
So corresponding mesh current equations are

$$\begin{cases} I_1R_1 + (I_1 - I_2)R_2 = V_s \quad \text{— (1)} \\ \cancel{(R_1+R_2)I_1 - I_2R_2} \\ -R_2I_1 + (R_2 + R_3 + R_4)I_2 = 0 \quad \text{— (2)} \end{cases}$$

By solving the equations, we can find the currents I_1 and I_2.

Mesh equations by inspection:-

- The mesh equations for a general planar network can be written by inspection without going through the detailed steps.



Matrix form of the mesh analysis by inspection is

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}_{(3\times3)} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}_{(3\times1)} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}_{(3\times1)}$$

So the mesh equations are

$$\left.\begin{aligned} R_{11} I_1 + R_{12} I_2 + R_{13} I_3 &= V_1 \\ R_{21} I_1 + R_{22} I_2 + R_{23} I_3 &= V_2 \\ R_{31} I_1 + R_{32} I_2 + R_{33} I_3 &= V_3 \end{aligned}\right\}$$

where R_{11}, R_{22} and R_{33} are the diagonal entries of the matrix and the rests are off-diagonal elements.

- For the diagonal elements, the value is given by the positive summation of all resistances connected to that particular loop.

~~So for,~~

So R_{11} = ~~R_1+R_2~~ sum of all the resistances connected to Loop-1 i.e. the sum of all the resistances through which I_1 passes.

$$\Rightarrow \boxed{R_{11} = R_1 + R_2}$$

R_{22} = sum of all the resistances connected to loop-2; i.e. sum of all the resistances through which I_2 passes.

$$\Rightarrow \boxed{R_{22} = R_2 + R_3}$$

R_{33} = Sum of all the resistances connected to Loop-3 i.e. through which I_3 passes.

$$\Rightarrow \boxed{R_{33} = R_4 + R_5}$$

- For off-diagonal elements, the resistance value is given by the sum of the resistances (i.e. mutual) common to ~~the~~ both the concerned Loops (mutual resistances). If the directions of currents passing through the common resistances are same, the mutual resistance will have positive sign; and if the direction of the currents passing through the common resistance ~~is~~ are opposite, then the mutual resistance will have a negative sign.

$R_{12} = R_{21} =$ Sum of resistances common to the Loop-1 and Loop-2 (mutual resistances) i.e. Sum of resistances common to Loop currents I_1 & I_2

$\Rightarrow$ $R_{12} = R_{21} = -R_2$; –ve sign because I_1 & I_2 are in opposite directions.

$R_{13} = R_{31} =$ Sum of resistances common to the Loop-1 and Loop-3 i.e. Sum of resistances common to Loop currents I_1 and I_3

$\Rightarrow$ $R_{13} = R_{31} = 0$; There is no common resistance between Loop-1 and Loop-3.

$R_{23} = R_{32} =$ Sum of resistances common to the Loop-2 and Loop-3 i.e. sum of resistances common to Loop currents I_2 and I_3.

$\Rightarrow$ $R_{23} = R_{32} = 0$; There is no common resistances through which both I_2 & I_3 flow.

* V_1, V_2, V_3 are the voltages which drives the loop. The sign will be positive if the direction of the current is same as the direction of source. If the current direction is opposite to the direction of the source, then the negative sign is used.

So

$V_1 = V_a$ is the voltage which drives Loop 1. Sign is positive as the direction of source is same as the direction of current.

$V_2 = -V_b$; is the voltage which drives Loop-2. Sign is negative because direction of current is opposite to the direction of source.

$V_3 = V_b$; is the voltage which drives Loop-3. Sign is positive because direction of current is same as the direction of source.

Putting these values in the mesh matrix ~~current equations~~.

$\cancel{(R_1+R_2)I_1 - R_2 I_3}$

$$\begin{bmatrix} (R_1+R_2) & -R_2 & 0 \\ -R_2 & (R_2+R_3) & 0 \\ 0 & 0 & R_4+R_5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_a \\ -V_b \\ V_b \end{bmatrix}$$

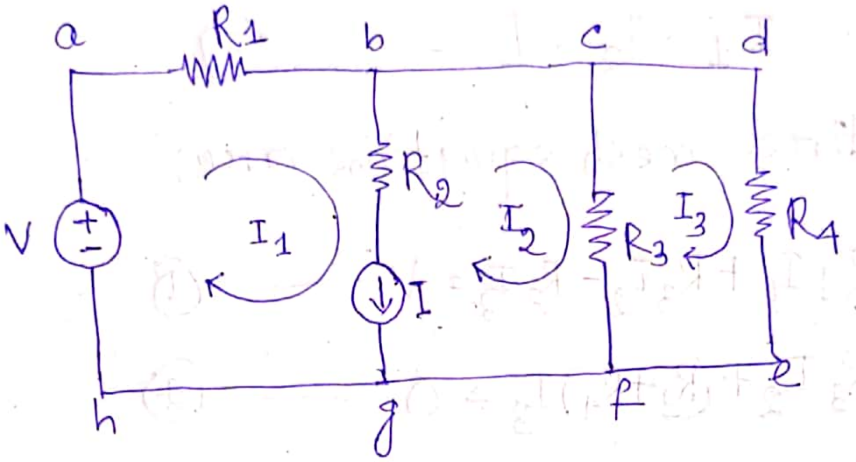
So the mesh current equations become

$$\left.\begin{aligned} (R_1+R_2)I_1 - R_2 I_2 &= V_a \\ -R_2 I_1 + (R_2+R_3)I_2 &= -V_b \\ (R_4+R_5)I_3 &= V_b \end{aligned}\right\}$$

Solving these equations, we can get the values of the mesh currents I_1, I_2 and I_3.

Supermesh Analysis:-

- A supermesh is consituted by two adjacent Loops that have a common current source.



Here, the current source I is common boundary for the two meshes 1 and 2. This current source creates a supermesh, which is nothing but a combination of meshes 1 and 2. In the above figure, supermesh is formed by the loop abcfgh.

Mesh equations

Applying KVL through the supermesh:

$$-V + I_1 R_1 + (I_2 - I_3) R_3 = 0$$

$$\Rightarrow R_1 I_1 + R_3 I_2 - R_3 I_3 = V \quad \text{———} \quad (1)$$

Applying KVL through mesh 3;

$$R_3 (I_3 - I_2) + R_4 I_3 = 0$$

$$\Rightarrow -R_3 I_2 + (R_3 + R_4) I_3 = 0 \quad \text{———} \quad (2)$$

Finally, the current I from the current source is equal to the difference between the two meshes currents i.e. $(I_1 - I_2)$.

$$I_1 - I_2 = I \text{ ——— } (3)$$

So the three mesh equations are;

$$\begin{cases} R_1 I_1 + R_3 I_2 - R_3 I_3 = V \text{ ——— } (1) \\ -R_3 I_2 + (R_3 + R_4) I_3 = 0 \text{ ——— } (2) \\ I_1 - I_2 = I \text{ ——— } (3) \end{cases}$$

Solving these above equations, we can get the three unknown currents I_1, I_2 and I_3.

Nodal Analysis :-

Node :- The common point where two or more elements are connected is called as node.

- When two elements are connected to a point, then the point is called as simple node.
- When three or more elements are connected to a common point, then the point is called as principal node.

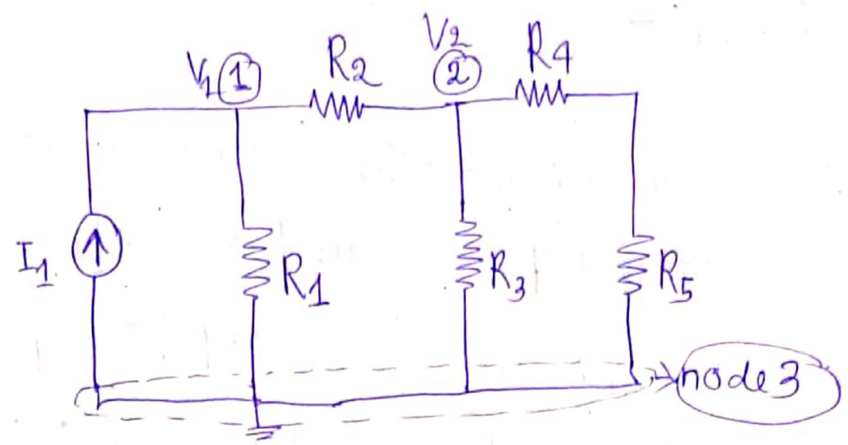
Procedures to solve nodal analysis problem:-

1) Identify the total no. of principal nodes in the given circuit.

2) Assign the voltage at each principal node by considering any one of the node as reference node which is connected to ground (i.e. zero potential)

3) Develop the KCL equation for each non-reference node.

4) Solve the KCL equations to get the node voltage.



In the figure, there are 3 nodes. i.e. ~~node~~ 1, 2 & 3.

Node 3 is assumed as ~~node~~ reference node.

Let V_1 and V_2 are the voltages at node-1 and node-2 respectively.

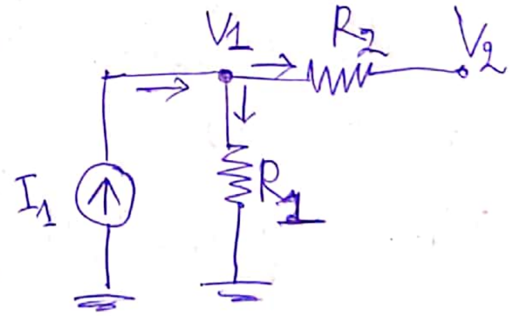
V_1 = Voltage at node 1 with respect to node-3

V_2 = Voltage at node 2 with respect to node 3

Applying KCL at node-1 :-

Total incoming current = Total outgoing current

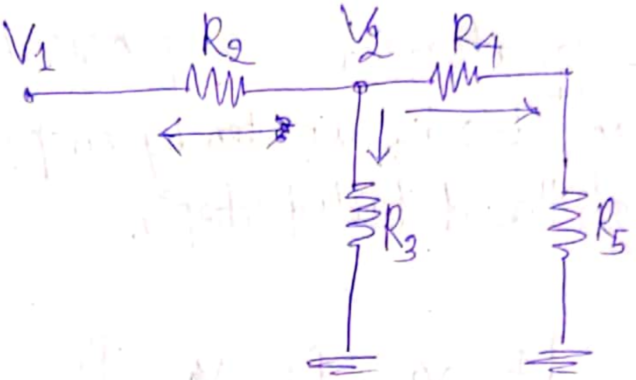
$$\Rightarrow \quad I_1 = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2}$$



$$\Rightarrow \boxed{\left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_1 - \frac{V_2}{R_2} = I_1} \quad (1)$$

** The unknown currents at node is taken outward from the node. (By convention)

Applying KCL at node-2:-



Total incoming current = Total outgoing current

$$\Rightarrow \quad 0 = \frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_2}{R_4 + R_5}$$

$$\Rightarrow \boxed{-\frac{V_1}{R_2} + \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4 + R_5}\right) V_2 = 0} \quad \text{— (2)}$$

Solving equation (1) & (2), we can get the voltage at each node i.e. V_1 and V_2.

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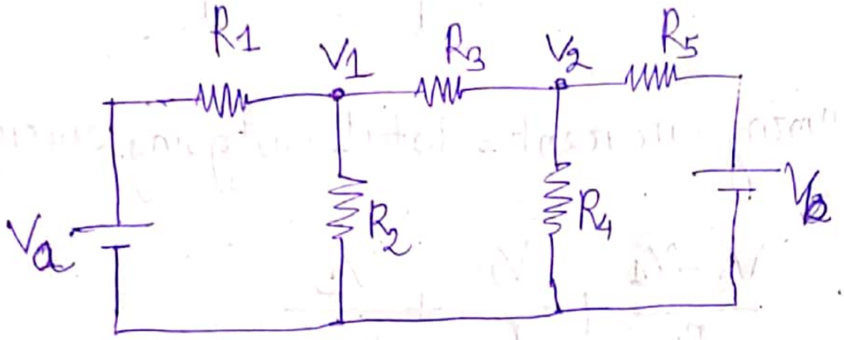
Nodal equations by inspection method :-

- The nodal equations for a general planar network can also be written by inspection, without going through the detailed steps.



Let V_1, V_2 are the node voltage of node-1 & node 2 respectively.

$$\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$V = IR$

$\Rightarrow \frac{V}{R} = I$

$\Rightarrow GV = I$

$\because \frac{1}{R} = G$

→ So the two nodal equations are

$$G_{11} V_1 + G_{12} V_2 = I_1 \qquad \text{—— (1)}$$

$$G_{21} V_1 + G_{22} V_2 = I_2 \qquad \text{—— (2)}$$

For diagonal elements:-

G_{11} = sum of the conductances connected to node-1.

$$\Rightarrow G_{11} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

G_{22} = sum of the conductances connected to node-2.

$$\Rightarrow G_{22} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}$$

For off-diagonal elements:-

G_{12} = sum of mutual conductances connected to node 1 and node 2.

$$\Rightarrow G_{12} = -\frac{1}{R_3}$$

G_{21} = sum of mutual conductances connected to node 2 and node 1

$$\Rightarrow G_{21} = -\frac{1}{R_3}$$

** Here, all the mutual conductances have negative sign.

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I_1 = sum of source current at node-1

$$\Rightarrow I_1 = \frac{V_a}{R_1}$$

I_2 = sum of source current at node 2.

$$\Rightarrow I_2 = \frac{V_b}{R_5}$$

** The current which drives into the node has positive sign, while the current that drives away from the node has negative sign.

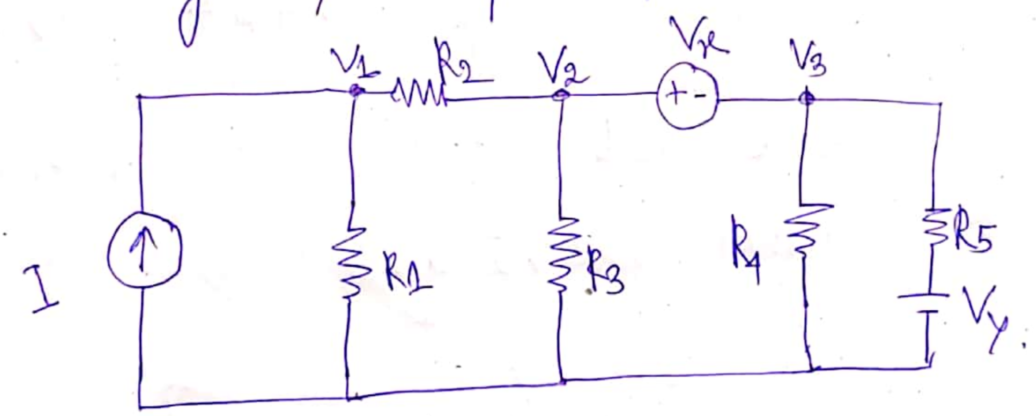
So the equation ① & ② becomes

$$\begin{cases} \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)V_1 - \left(\frac{1}{R_3}\right)V_2 = \frac{V_a}{R_1} \\ \left(-\frac{1}{R_3}\right)V_1 + \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right)V_2 = \frac{V_b}{R_5} \end{cases}$$

Solving these above two equations, we can get the value of V_1 and V_2.

Supernode Analysis:-

- In any electrical network, if there is a common ideal voltage source in between the two nodes, it becomes slightly difficult to apply nodal analysis, then supernode method is used.
- In this method, two nodes that are having common ideal voltage source are reduced to a single node and then KCL is applied to both the nodes simultaneously, thereby writting only one equation.



At node-1

$$I = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2}$$

$$\Rightarrow \boxed{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)V_1 - \frac{V_2}{R_2} = I} \quad \text{——} \ (1)$$

*(I is constant)

Node 2 and Node 3 are having a common ideal voltage source V_x. So supernode method is used i.e. writting the combined KCL equations for node 2 and node 3.

$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_3}{R_4} + \frac{V_3 - V_y}{R_5} = 0$$

$$\Rightarrow \boxed{\left(-\frac{1}{R_2}\right)V_1 + \left(\frac{1}{R_2} + \frac{1}{R_3}\right)V_2 + \left(\frac{1}{R_4} + \frac{1}{R_5}\right)V_3 = \frac{V_y}{R_5}}$$

* (V_y is constant) i.e. given. (2)

- Define the ideal voltage source in terms of unknown node voltages

$$\boxed{V_2 - V_3 = V_x} \quad \text{——— (3)}$$

* (V_x is constant) i.e. given.

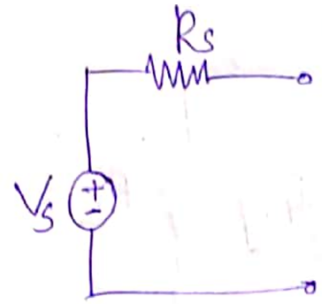
Solving the above three equations, we get can the value of V_1, V_2 and V_3.

*

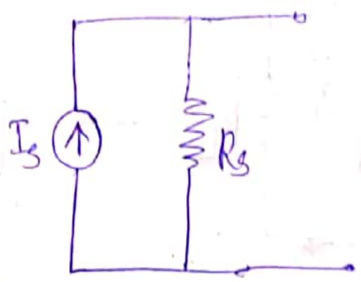
Source transformation technique :-

- It is the simplified technique which eliminates the extranode present in a network.

- Any practical voltage source consists of an ideal voltage source in series with an internal resistance. Similarly, a practical current source consists of an ideal current source in parallel with an internal resistance.



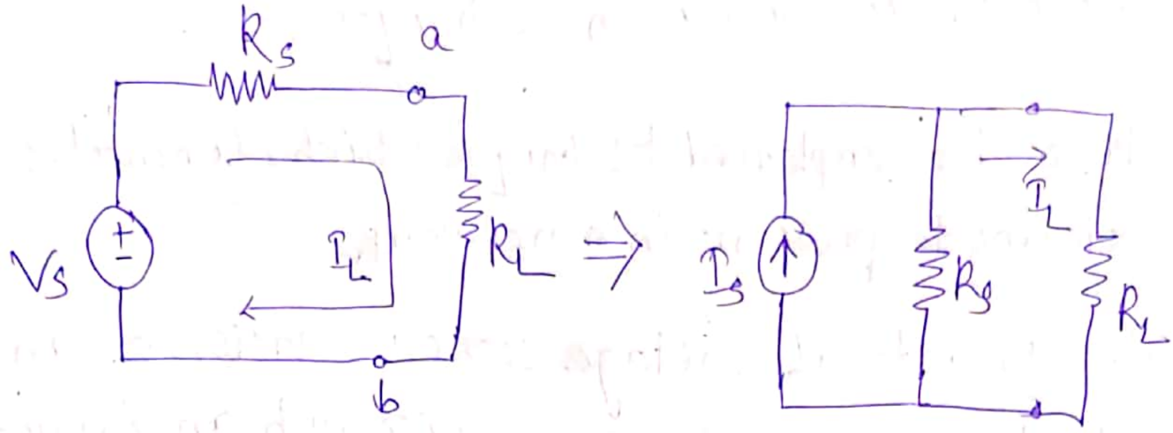
(Practical voltage source)



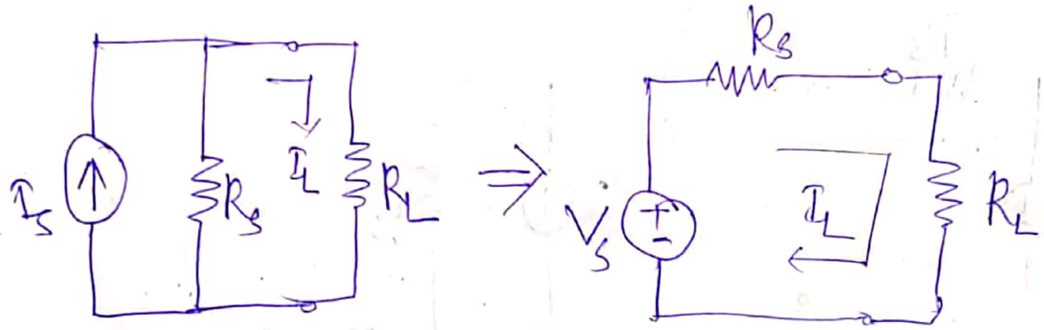
(Practical current source)

- By applying source transformation technique, a practical voltage source can be converted to a practical current source and vice versa.

- Source transformation technique is only applicable to practical sources and it is impossible to convert an ideal voltage source into an ideal current source, since there is no resistance in the circuit.



$$\boxed{I_s = \frac{V_s}{R_s}}$$



$$\boxed{V_s = I_s R_s}$$