Control System

System:- The physical arrangement of components in planned or sequential manner in order to perform a specific function or task is said to be system.

Ex:- mobile, aircraft, bike, fan etc.

Control system:- If the output of the system can be varied by controlling it's input, then it is said to be control system.

Ex:- AC, traffic, light, fan etc.

Control system are classified into 2 types:-

1) Open loop control system
2) Closed loop control system.

Open Loop control system:- If the output of the system is not taken into consideration for controlling the input (indirectly output), then it is called open loop control system.

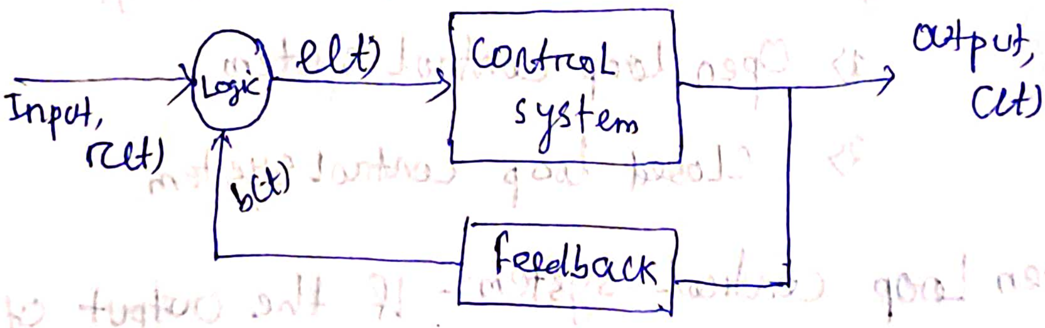
$r(t) \rightarrow$ input / excitation

$c(t) \rightarrow$ output / Response

Ex: Fan, traffic Light without sensors.

Closed Loop control system:-

If the o/p of the system is taken into consideration for controlling the input (indirectly output) then it is called closed loop control system.

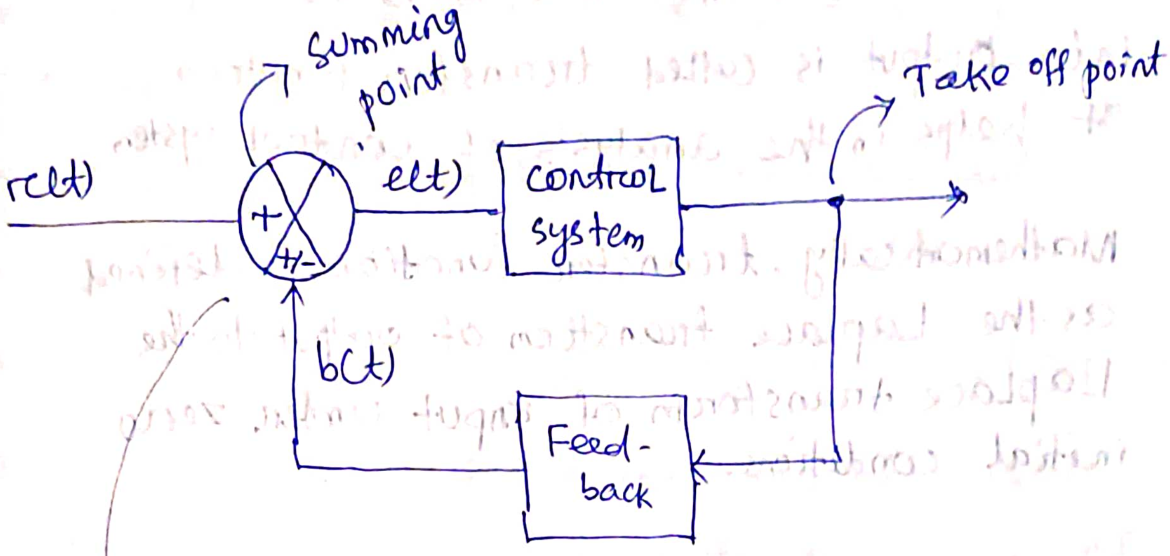


$b(t) \rightarrow$ base signal / feedback signal

$e(t) \rightarrow$ error signal.

Ex: Traffic light with sensors, AC, missile, satellite, Launching.

General representation of Closed Loop System:-



\+ → Positive feedback

\- → Negative feedback

In general, closed loop systems are negative feedback system.

Differences betn Open loop & Closed Loop system:-

O.L	C.L.
i) Simple to design & construct	i) Complex to design and construct.
ii) Design cost is Less.	ii) Construction cost is high.
iii) More stable compared to CL.	iii) Less ~~than~~ stable compared to O.L.
iv) Less accurate.	iv) More accurate.
v) can be easily affected by noise.	v) Noise can be eliminated or reduced.
vi) B.W. is less.	vi) B.W. is more
vii) High gain	vii) Less gain

Transfer function:-

The function which transforms the input into output is called transfer function. It helps in the analysis of control system.

Mathematically, transfer function is defined as the Laplace transform of output to the Laplace transform of input under zero initial conditions.

$$\text{Transfer function} = \left.\frac{L.T.\{\text{output}\}}{L.T.\{\text{input}\}}\right|_{\text{initial conditions}=0}$$

$$= \frac{L.T.\{C(t)\}}{L.T.\{r(t)\}} = \frac{C(s)}{R(s)}$$

$$\Rightarrow \quad \cancel{T.F.} \quad T.F = \frac{C(s)}{R(s)}.$$

$$\therefore \quad G(s) = \frac{C(s)}{R(s)}$$

Limitations of transfer functions:-

i) It is defined for single-i/p-single-o/p systems only.

ii) It is applicable to linear time invariant systems only.

iii) It is defined under zero initial conditions only.

iv) It can't predict the internal behaviour of the component but gives the properties of a system as a whole.

Transfer function is method is highly useful in the preliminary design and basic insight into the control system.

Note:

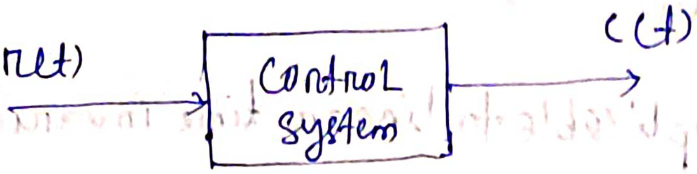
the transfer function of the system depends on the system design and components present inside the system; it is independent of i/p and o/p of the system.

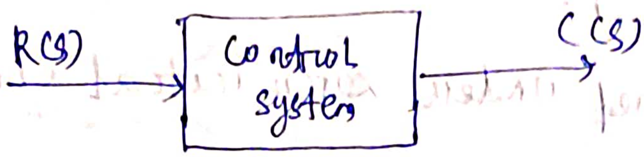
* The actual equation which represents T.F. is

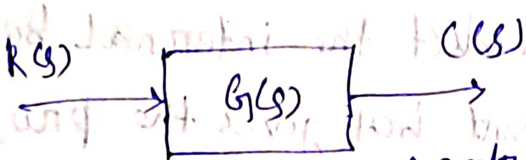
L.T. {output} = T.F. × L.T. {input}

i.e. output of the system depends on both T.F. and input of the system.

Open Loop control system:-

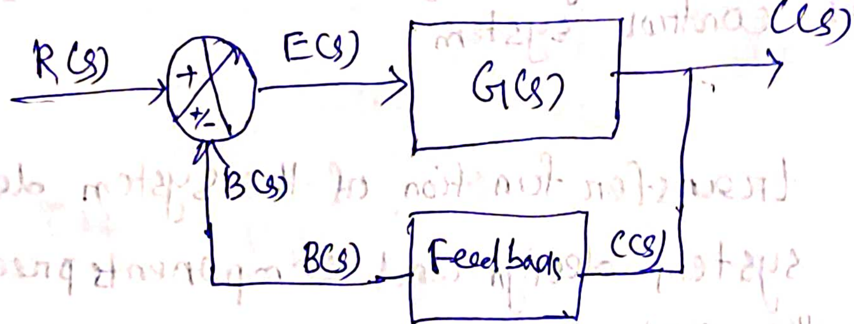






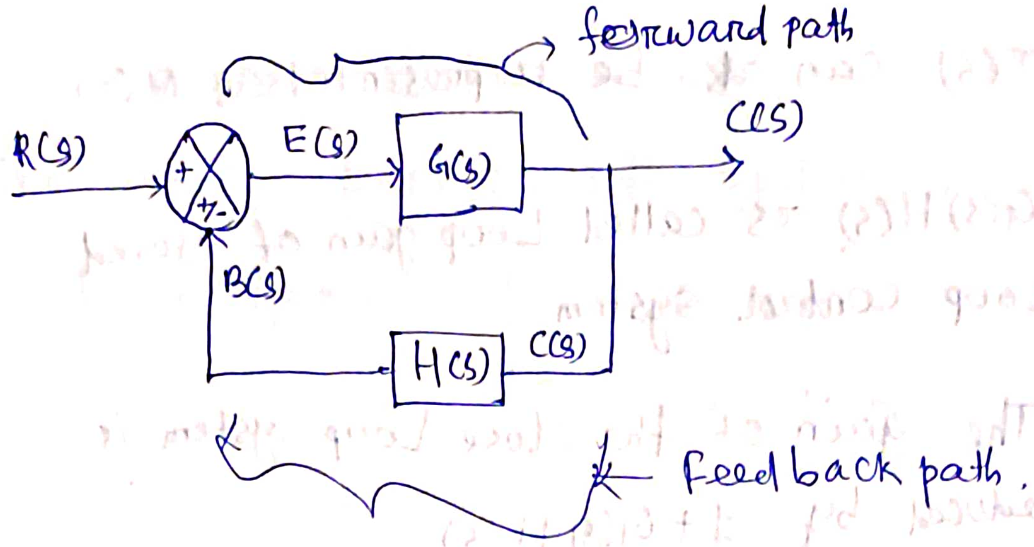
G(s) → Open Loop ~~transfer function~~ control system.

Close Loop control system:-



$$T.F._{(feedback)} = \frac{\cancel{B(s)}}{\cancel{H(s)}} \quad \frac{B(s)}{C(s)}$$

$$= H(s)$$



$$E(s) = R(s) - B(s)$$

$$G(s) = \frac{C(s)}{E(s)}$$

$$C(s) = E(s)\,G(s)$$

$$= [R(s) - B(s)]\,G(s)$$

$$\Rightarrow C(s) = [R(s) - C(s)H(s)]\,G(s)$$

$$\Rightarrow C(s)[1 + G(s)H(s)] = R(s)\,G(s)$$

$$\Rightarrow \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$\Rightarrow \boxed{T(s) = \frac{G(s)}{1 + G(s)H(s)}}$$

Close Loop T.F. = $T(s) = \dfrac{G(s)}{1 \pm G(s)H(s)}$

$+ \rightarrow$ -ve feedback

$- \rightarrow$ +ve feedback.

→ $T(s)$ can also be represented as $M(s)$.

→ $G(s)H(s)$ is called Loop gain of closed Loop control system.

→ The gain of the close Loop system is reduced by $1+G(s)H(s)$.

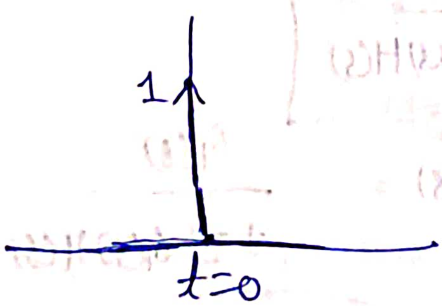
*/ Close Loop control system is Less sensitive to changes in $G(s)$ but it is more sensitive to changes in $H(s)$.

Test Signals:-

Test signals ~~is~~ are the inputs applied to the control system to obtain its transfer function.

Unit Impulse input:-

$$\delta(t) = 1;\ t=0$$
$$= 0;\ t\neq 0$$



$LT\{\delta(t)\} = 1$

The impulse response of the system is

$LT\{output\} = T.F. \times LT.\{i/p\}$

$= T.F. \times LT.\{\delta(t)\}$

$= T.F. \times 1$

$\Rightarrow LT.\{O/p\} = T.F.$ i.e. $C(s) = T.F$

impulse response

- The response of the system when the i/p is impulse is called impulse response. It is denoted by I.R.

- Impulse response is the transfer function of the system in time domain.

T.F. → 's' domain

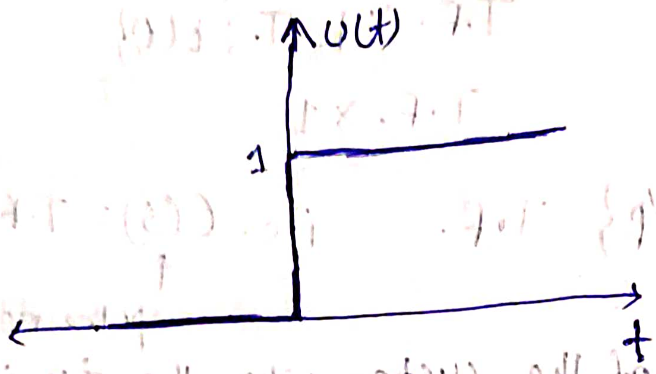
I.R → 't' domain

$\Rightarrow$ $\boxed{T.F. = L.T.\{I.R\}}$

- It is not possible to apply impulse input practically. It's only theoretical i/p.

Unit step input:-

$$u(t) = 1 ; \ t \geq 0$$
$$= 0 ; \ t < 0$$



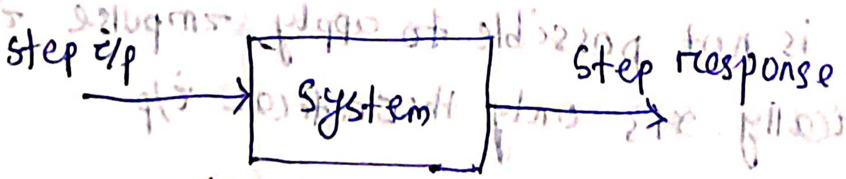
* $LT\{u(t)\} = \frac{1}{s}$

→ $\frac{d\,u(t)}{dt}$ is the impulse signal i.e.

$$\delta(t) = \frac{du(t)}{dt}$$

→ Therefore by applying step input we can obtain transfer function of the system as it is related to $\delta(t)$.

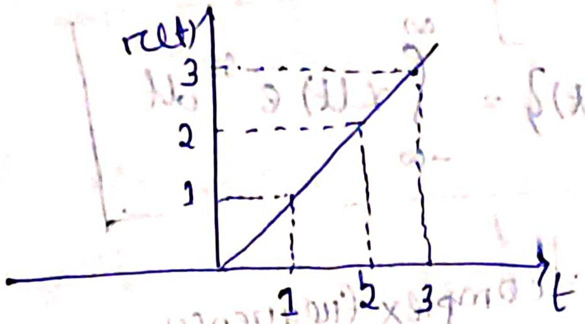
- Impulse response = $\frac{d}{dt}\{\text{step response}\}$



- We can apply step signal practically.

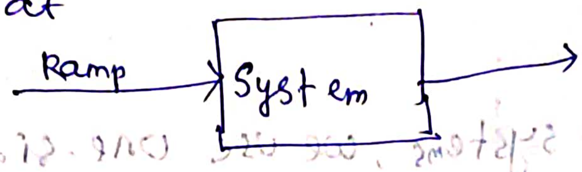
Unit ramp i/p :-

$$r(t) = t \ ; \ t \geq 0$$
$$= 0 \ ; \ t < 0$$



$$\therefore \ L.T.\{r(t)\} = \frac{1}{s^2}$$

$$\frac{dr(t)}{dt} = u(t)$$



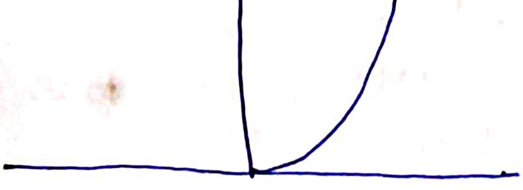
$$IR = \frac{d}{dt}\{\text{Step response}\}$$
$$= \frac{d}{dt}\left\{\frac{d}{dt}(\text{Ramp response})\right\}$$

$$\Rightarrow I.R = \frac{d^2}{dt^2}\{\text{Ramp response}\}$$

→ Ramp input can be applied practically.

Unit parabolic i/p :-

$$P(t) = \frac{1}{2}t^2 \ ; \ t \geq 0$$
$$= 0 \ ; \ t < 0$$



$$L.T.\{P(t)\} = \frac{1}{s^3}$$

Laplace Transform:-

For a signal $x(t)$, the Laplace transform is defined as

$$\boxed{LT\{x(t)\} = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt}$$

where 's' is the complex frequency,

$$s = \delta + j\omega.$$

δ gives the Region of Convergence of Laplace Transform.

- For control systems, we use one-sided LT.

i.e.
$$\boxed{X(s) = \int_{0}^{\infty} x(t)\, e^{-st}\, dt}$$

- Laplace transform converts time domain signal to complex frequency domain, s.
- If $s = j\omega$, ($\delta = 0$), Laplace transform becomes Fourier transform.

1. $LT\{\delta(t)\} = 1$
2. $L.T.\{u(t)\} = \frac{1}{s}$
3. $L.T.\{t^n\} = \frac{n!}{s^{n+1}}$

4. $LT\{e^{-at}u(t)\} = \frac{1}{s+a}$.

5. $LT\{\cos\omega t\} = \frac{s}{s^2+\omega^2}$

6. $L.T.\{\sin\omega t\} = \frac{\omega}{s^2+\omega^2}$

7. If $x(t) \rightleftharpoons X(s)$

$e^{-at}x(t) \rightleftharpoons X(s+a)$ [frequency shifting]

8. If $x(t) \rightleftharpoons X(s)$

$x(t-t_0) \rightleftharpoons X(s)e^{-st_0}$

9. If $x(t) \rightleftharpoons X(s)$, then $\frac{dx(t)}{dt} \rightleftharpoons sX(s)$,

$\int x(t)\,dt \rightleftharpoons \frac{1}{s}X(s)$

10. $L.T.\{e^{-at}\cos\omega t\} = \frac{s+a}{(s+a)^2+\omega^2}$

11. $LT\{e^{-at}\sin\omega t\} = \frac{\omega}{(s+a)^2+\omega^2}$

Initial value theorem:-

If $x(t) \rightleftharpoons X(s)$, then
initial value of $x(t)$

$$x(0) = \underset{t\to 0}{Lt}\ x(t) = \lim_{s\to\infty} s.X(s)$$

e.g.

$$X(s) = \frac{2s+3}{s(5s+6)}$$

$$X(0) = \underset{s\to\infty}{Lt}\ s\cdot\frac{2s+3}{s(5s+6)} = \frac{2}{5}$$

Final value theorem:-

If $x(t) \rightleftharpoons X(s)$, then
final value of $x(t)$

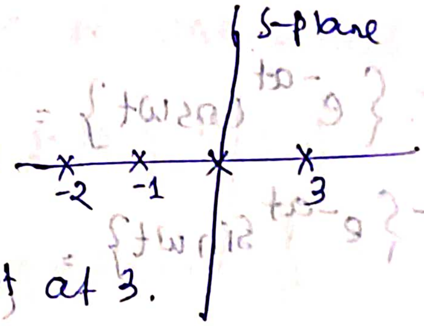
$$x(\infty) = \lim_{t \to \infty} x(t) = \lim_{s \to 0} s\,X(s)$$

$$\boxed{X(\infty) = \lim_{s \to 0} s \cdot X(s)}$$

Note:-

When the poles of the function are present on imaginary axis or on the rhs of s-plane, we can't apply final value theorem.

i) $$X(s) = \frac{2(s+1)}{s(s+2)(s-3)}$$

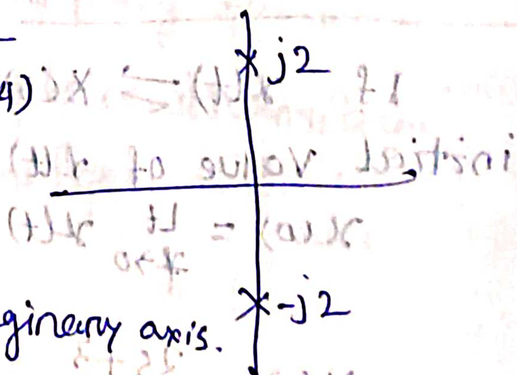


poles = $0, -2, +3$

Here a pole is present at 3.

So final value theorem can't be applied.

ii) $$X(s) = \frac{1}{s(s^2+4)}$$



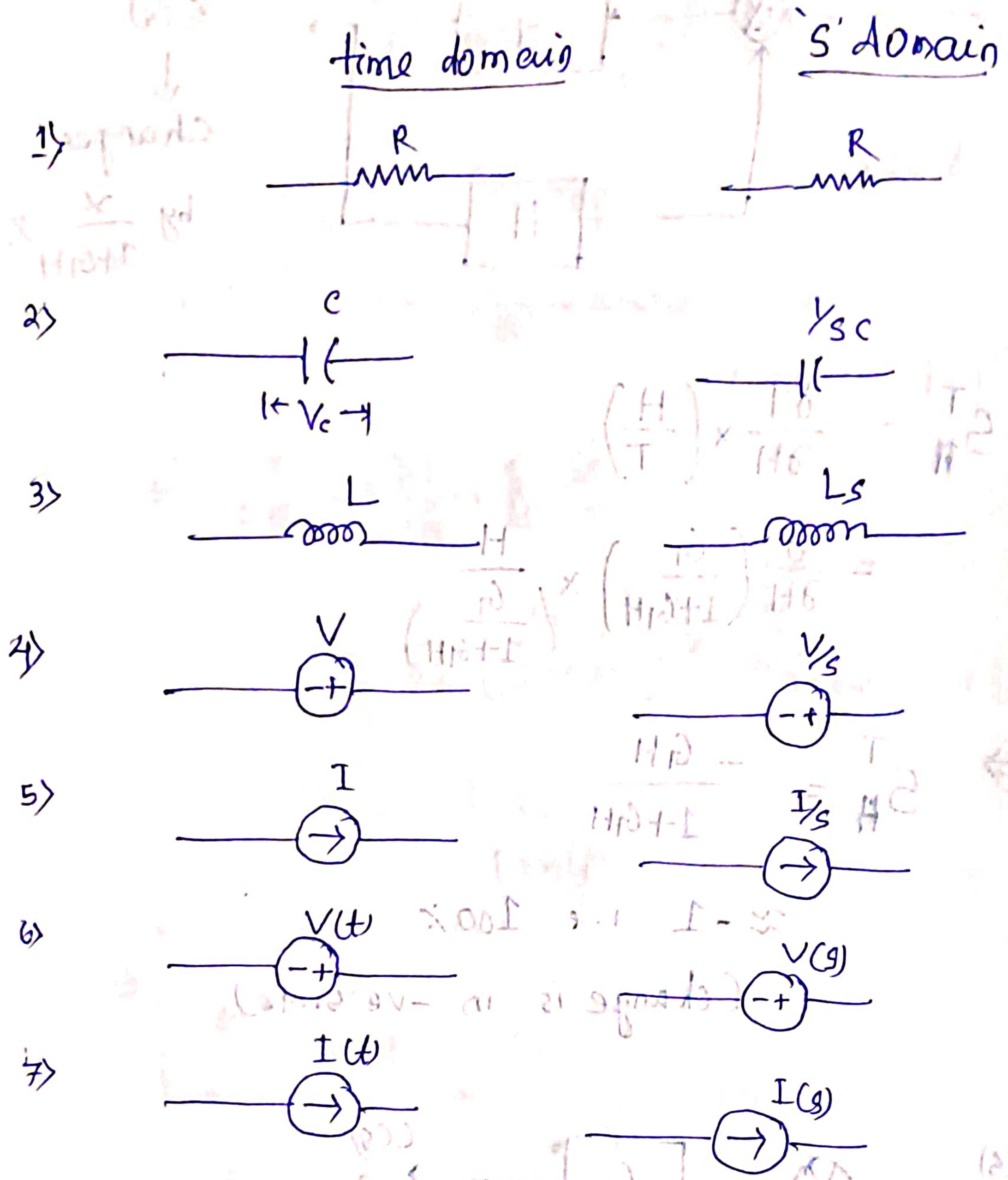
Poles = $0, \pm j2$.

Poles present on imaginary axis.
Hence final value theorem can't be applied.

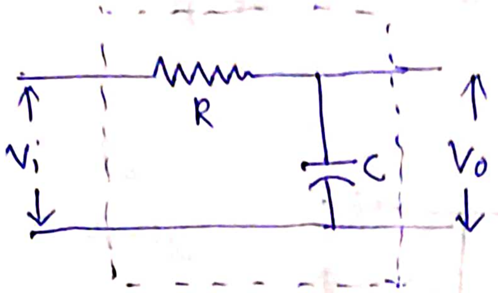
iii) $$X(s) = \frac{2}{s(s+3)}, \quad \text{Poles} = 0, -3$$

$$X(\infty) = \lim_{s \to 0} s \cdot \frac{2}{s \cdot (s+3)} = \frac{2}{3}.$$

Transfer function of electrical system:-



Q) Obtain the TF of the electrical n/w shown in figure:-



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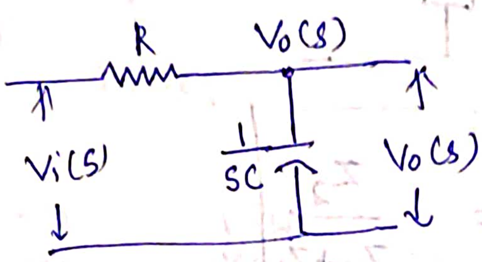


Applying KCL at $V_o(s)$

$$\frac{V_o(s) - V_i(s)}{R} + \frac{V_o(s)}{1/sC} = 0$$

$$\Rightarrow V_o(s)\left(\frac{1}{R} + \frac{1}{1/sC}\right) = \frac{V_i(s)}{R}$$

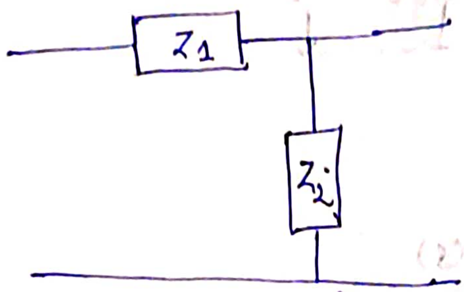
$$\Rightarrow \left(\frac{1+sRC}{R}\right) V_o(s) = \frac{V_i(s)}{R}$$

$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{1}{1+sRC}$$

Notes

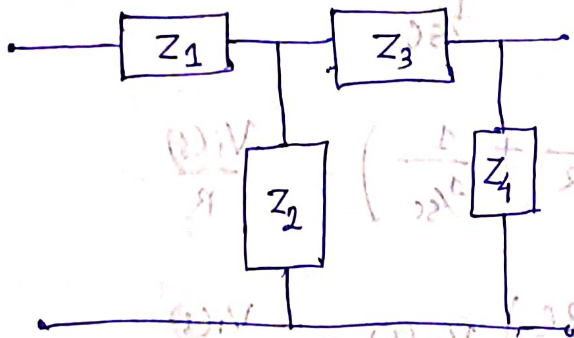
for the electrical network shown in the figure

1)



$$\boxed{T.F = \frac{Z_2}{Z_1+Z_2}}$$

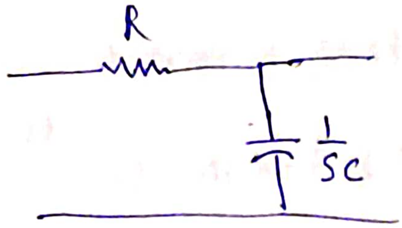
2)



$$\boxed{T.F. = \frac{Z_2 Z_4}{Z_1(Z_2+Z_3+Z_4)+Z_2(Z_3+Z_4)}}$$

Ex:

i)



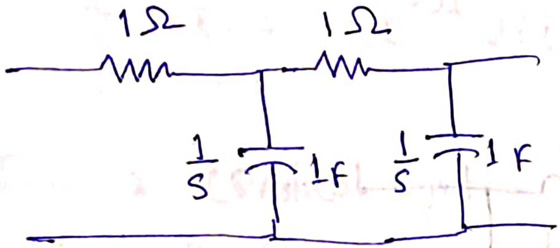
$Z_1 = R$

$Z_2 = \frac{1}{sc}$

$$TF = \frac{Z_2}{Z_1+Z_2} = \frac{\frac{1}{sc}}{R+\frac{1}{sc}}$$

$$\Rightarrow T.F = \frac{1}{1+sRC}$$

ii)



$Z_1 = 1\,\Omega$

$Z_2 = \frac{1}{s}\,\Omega$

$Z_3 = 1\,\Omega$

$Z_4 = \frac{1}{s}\,\Omega$

$$TF = \frac{Z_2 Z_4}{Z_1(Z_2+Z_3+Z_4)+Z_2(Z_3+Z_4)}$$

$$= \frac{\frac{1}{s}\times\frac{1}{s}}{1\left(\frac{1}{s}+1+\frac{1}{s}\right)+\frac{1}{s}\left(1+\frac{1}{s}\right)} = \frac{\frac{1}{s^2}}{\frac{2+s}{s}+\frac{s+1}{s^2}}$$

$$\Rightarrow T.F. = \frac{1}{s^2+3s+1}$$

Q) For a control system if unit step response is $e^{-2t}u(t)$. What is the impulse response of the system.

Soln :- Impulse response $= \frac{d}{dt}$ (step response)

$$= \frac{d}{dt}\left(e^{-2t}u(t)\right)$$

$$= -2 \cdot e^{-2t}u(t)$$

$$\Rightarrow I.R. = -2e^{-2t}u(t).$$

Q) For a control system if the unit impulse response is $3e^{-3t}u(t)$. What is the dc gain of the system.

Soln :-

$$I.R = 3e^{-3t}u(t)$$

$$T.F = LT\{I.R\} = \frac{3}{s+3}$$

For dc gain, $s=0$

$$\text{Gain}(T.F) = \frac{3}{0+3} = 1.$$

Q) For a control system if the O.L.T.F. $G(s) = \frac{20}{s^2}$ and feedback function $H(s) = 2(s+2)$. Determine the steady state value (final value) when the input is $u(t)$.

Soln:-
$$G(s) = \frac{20}{s^2} \;,\; H(s) = 2(s+2)$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$\Rightarrow C(s) = R(s) \cdot \frac{G(s)}{1+G(s)H(s)}$$

$$= \frac{1}{s} \cdot \frac{20/s^2}{1+\frac{20}{s^2} \times 2(s+2)}$$

$$\Rightarrow C(s) = \frac{1}{s} \cdot \left(\frac{20}{s^2+40s+80}\right)$$

$$\Rightarrow C(\infty) = \lim_{s \to 0} s \times \frac{1}{s}\left(\frac{20}{s^2+40s+80}\right) = \frac{1}{4}$$

$$\Rightarrow C(\infty) = \frac{1}{4}.$$

Q) If the impulse response of a control system is $e^{-2t}u(t)$. What is the response of the system when the input is $e^{-3t}u(t)$.

Soln.

$$I.R = e^{-2t}u(t).$$

$$i/p = e^{-3t}u(t)$$

$$o/p = ?$$

$\because r(t) = e^{-3t} u(t) \Rightarrow R(s) = \frac{1}{s+3}$

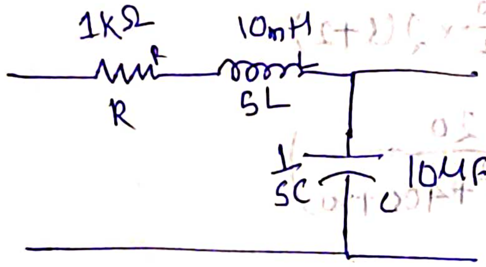
$L.T.\{output\} = T.F. \times L.T.\{input\}$

$= \frac{1}{s+2} \times \frac{1}{s+3}$

$\Rightarrow C(s) = \frac{1}{s+2} - \frac{1}{s+3}$

$\Rightarrow$ output, $c(t) = (e^{-2t} - e^{-3t}) u(t)$

Qs T.F. of the electrical network shown in the figure is



$TF = \frac{\frac{1}{sC}}{\frac{1}{sC} + R + sL}$

$\Rightarrow TF = \frac{1}{s^2 LC + sRC + 1}$

$= \frac{1}{s^2(10\times10^{-3} \times 10\times10^{-6}) + s(1\times10^{3} \times 10\times10^{-6}) + 1}$

Q) The impulse response of a system is $(e^{-t} - e^{-2t})\, u(t)$ find the T.F. of the system?

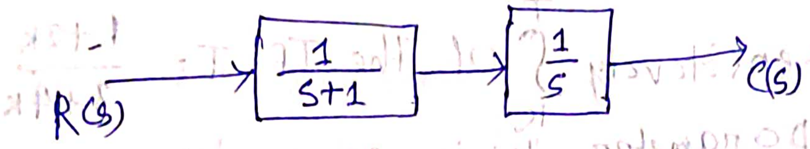
$$T.F. = L.T.\{\text{impulse response}\}$$

$$= L.T.\{e^{-t} - e^{-2t}\}$$

$$= \frac{1}{s+1} - \frac{1}{s+2}$$

$$\Rightarrow T.F = \frac{1}{(s+1)(s+2)}$$

Q) What is the unit impulse response of the system shown in figure at $t \geq 0$.



Soln:-

$$\frac{C(s)}{R(s)} = \frac{1}{(s+1)} \times \frac{1}{s}$$

$$\Rightarrow T.F = \frac{1}{s(s+1)}$$

$$I.R = L^{-1}\{TF\}$$

$$= L^{-1}\left\{\frac{1}{s(s+1)}\right\}$$

$$= L^{-1}\left\{\frac{1}{s} - \frac{1}{s+1}\right\}$$

$$= (t - e^{-t})\, u(t)$$

$$\Rightarrow I.R = (t - e^{-t})\, u(t).$$