

UNIT-1 UNITS & DIMENSIONS

Physical quantity is a physical property that can be quantified and can be measured using numbers. Ex → mass, amount of substance, length, time, temperature, electric current, light intensity, force, velocity, density and many more.

Fundamental quantity → The base quantity which cannot be derived from each other, nor can they be resolved into anything simpler or basic. The units used to measure fundamental quantities are called fundamental units. Ex → m, kg, s, K, A, mol, °C.

Derived units → Derived quantities are those whose definitions are based on other physical quantities; eg → speed, area, density,

The branch of science which deals with the study of matter. Physics is the branch of that natural science which deals with the physical world and the principle governing its behaviour. The term of quantity which is used as a standard measurement is called its units.

Measurement is said to be complete if we know its unit and how many times the unit is contained in its measurement.

There are two types of unit.

Fundamental unit → The unit of length, mass and time are called fundamental unit and the quantity that is length, mass and time are called fundamental quantity at physics.

Derived unit → The unit of other physical except mass, length and time can be derived by taking the help of the unit of fundamental unit and is called derived unit.
(Velocity, acceleration)

Dimension → The dimension of a derived physical quantity may be defined as the powers to which its base units must be raised to represent it completely.

Dimensional formula → A dimensional formula is an expression which shows how (with what powers) and which of its base unit or fundamental units enter into the units of a physical quantity.

Dimensional equation → The equation written in the square bracket and by expressing the values of the base dimensions
For ex: $[M^1 L^1 T^{-2}]$

The bracket $[]$ used in the equation signify that the equation represents the qualitative nature of area only. It does not give any information about the magnitude of area.

Principle of Homogeneity → The dimensional formula of every term on the two sides of a correct relation must be same.

Convert 1 J into erg

$$\begin{aligned} 1 \text{ J} &= \text{Nm}^2 \\ &= 1 \text{ kg} \times 100 \text{ cm} \times 100 \text{ cm} \\ &= 1000 \text{ g} \times 100 \text{ cm} \times 100 \text{ cm} \\ &= 10^7 \text{ erg} \end{aligned}$$

At M.K.S (system)

$$M = 1 \text{ kg}$$

$$L = 1 \text{ m}$$

$$T = 1 \text{ sec}$$

$$n_1 = 1$$

Work

$$M = (1)$$

$$L = (2)$$

$$T = (-2)$$

C.G.S

$$M = 1 \text{ g}$$

$$L = 1 \text{ cm}$$

$$T_2 = 1 \text{ sec}$$

$$n_2 = 2$$

There are three different systems to measure the fundamental quantity.

- ① C.G.S system of unit
- ② M.K.S "
- ③ F.P.S "

C.G.S \rightarrow This system is also called french system and cm is the unit of length, gm is the unit of mass, t-sec is the unit of time. Also known as Gaussian system.

F.P.S \rightarrow This system is known as British system foot is the unit of length, pound is the unit of force and sec is the unit of time.

M.K.S \rightarrow This system is known as system of International the S.I system m is the unit of length, kg is the unit of mass and sec is the unit of time.

Fundamental quantity	C.G.S system	M.K.S system	F.P.S system
Length	cm	m	ft (foot)
Mass	gm	kg	lb (pound)
Time	second	second	second

Uses of Dimensional Analysis

There are three uses which have been put for dimensional analysis

- ① To convert the values of a physical quantity from one system to another
- ② To check the correctness of a given relation.
- ③ To derive a relation between various physical quantities

The use depends upon the principle of homogeneity →
 The dimensional formula of every term on the two sides
 of a correct relation must be same.

① Convert 1 J into erg

$$\begin{aligned} 1 \text{ J} &= \text{Nm}^2 \\ &= 1 \text{ kg} \times 100 \text{ cm} \times 100 \text{ cm} \\ &= 1000 \text{ g} \times 100 \text{ cm} \times 100 \text{ cm} = 10^7 \text{ erg} \end{aligned}$$

M.K.S (V.K)

$$M_1 = 1 \text{ kg}$$

$$L_1 = 1 \text{ m}$$

$$T_1 = 1 \text{ s}$$

$$n_1 = 1$$

Block $[M^a L^b T^c]$

$$M = 1 = a$$

$$L = 2 = b$$

$$T = -2 = c$$

C.G.S (V. Unknown)

$$M_2 = 1 \text{ g}$$

$$L_2 = 1 \text{ cm}$$

$$T_2 = 1 \text{ s}$$

$$n_2 = ?$$

In system 1 the physical quantity is $= n_1 [M_1^a L_1^b T_1^c]$

In system 2 the physical quantity is $= n_2 [M_2^a L_2^b T_2^c]$

Since the quantity is same so $n_1 [M_1^a L_1^b T_1^c] = n_2 [M_2^a L_2^b T_2^c]$

$$\Rightarrow n_2 = n_1 \left[\frac{M_1}{M_2} \right]^a \left[\frac{L_1}{L_2} \right]^b \left[\frac{T_1}{T_2} \right]^c$$

substituting the values as given in the problem

$$\Rightarrow n_2 = 1 \left[\frac{1 \text{ kg}}{1 \text{ g}} \right]^1 \left[\frac{1 \text{ m}}{1 \text{ cm}} \right]^2 \left[\frac{1 \text{ s}}{1 \text{ s}} \right]^{-2}$$

$$= 1 \times \left[\frac{1000 \text{ g}}{1 \text{ g}} \right]^1 \left[\frac{100 \text{ cm}}{1 \text{ cm}} \right]^2 \times 1 \Rightarrow 1 \text{ J} = 10^7 \text{ erg}$$

② To check the correctness of a given relation

③ To derive a relation between various physical quantities

UNIT-2 Scalars & Vectors

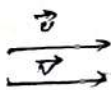
Scalar \rightarrow The quantity which has only magnitude is called scalar quantity. Ex: Mass, time, temperature etc.

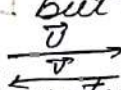
Vector \rightarrow The quantity which has magnitude as well as direction is called vector quantity. Ex: displacement.

The representation of the vector is the arrowhead symbol
i.e; \vec{x}

Terms connected with vectors

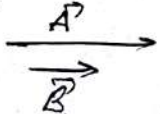
(i) Null vector $(\vec{0})$ \rightarrow The vector which has zero magnitude and an arbitrary direction.

(ii) Equal vector \rightarrow Two vectors are said to be equal if they possess the same magnitude and direction.
 (The angle between two equal vectors is 0°)

(iii) Opposite vector \rightarrow Two vectors are said to be opposite if they possess same magnitude but in different direction.
 (The angle between the opposite vectors of two sides is 180°)

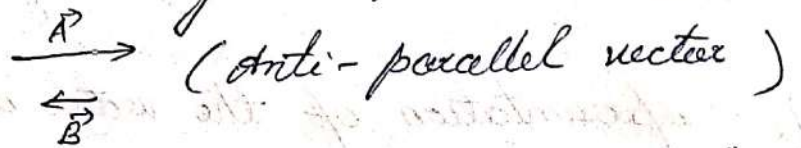
(iv) Co-initial vector \rightarrow A number of vectors having a common initial point are called co-initial vectors.



(v) Co-linear vectors \rightarrow The two vectors are said to be co-linear if they have same line of action. They are two types (a) Parallel vectors \rightarrow Two vectors are said to be parallel if they are acting in the same direction irrespective of their magnitude
 (Parallel vectors)
The angle between two parallel vectors is 0°

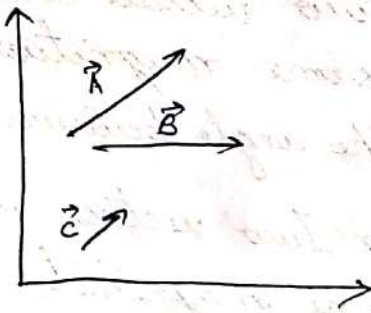
It is to be noted that all equal vectors are parallel but all parallel vectors are not equal.

(vi) Anti-parallel vectors \rightarrow Two vectors are said to be anti-parallel if they are acting in opposite direction.



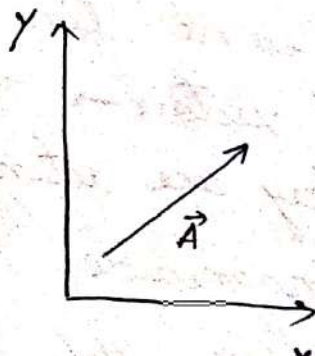
The angle between two anti-parallel vectors is 180° . All opposite vectors are anti-parallel but all anti-parallel vectors are not opposite.

(vii) Co-planar vectors \rightarrow The number of vectors lying on a common plane are called co-planar vectors.



(viii) Localized vector / Fixed vector \rightarrow A vector whose initial point is fixed is called a localized vector or fixed vector.

(ix) Non-localized vector / Free vector \rightarrow A vector whose initial point is not fixed is called non-localized vector.



Scalar multiplication of a vector

When a scalar is multiplied with a vector then the result is vector quantity. The magnitude of the resultant vector is equal to the scalar time of the magnitude of the given vector and the direction of the resultant vector is same as the direction of the given vector.

Let $\vec{A} = 15 \text{ km}$ Towards east

$$m = 10$$

Then, $m(\vec{A}) = 10 \times 15 \text{ km}$ towards east

The angle between \vec{A} & \vec{R} is 0°

i.e. $\left(\frac{150 \text{ km}}{SQ} \text{ Towards east} \right) \vee Q$.

Addition of vectors

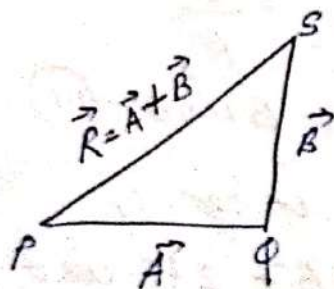
Scalar quantities can be added by simple algebra. As example when a 5kg of rice is added to 3kg of rice it becomes 8kg of rice.

But vector quantities cannot be added by simple algebra.

There are some special laws.

(i) Triangle law of vector addition

statement \rightarrow If two vectors (\vec{A} & \vec{B}) are represented in magnitude and direction by the two sides of a triangle taken in same order then the third side of the triangle will represent the resultant vector in magnitude and direction taken in opposite order.



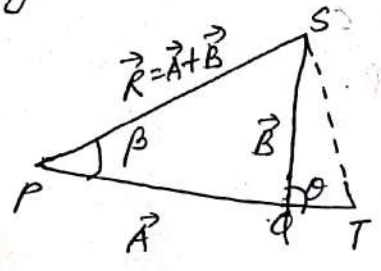
It can be proved by ^{Fig: (1)} two methods

① Graphical method \rightarrow In the ΔPQS $\vec{PQ} = \vec{A}$
 PQ vector is equal to \vec{A} $\vec{QS} = \vec{B}$
 $\vec{PS} = \vec{R}$

Here \vec{A} & \vec{B} are in the same order.

According to the triangle law of vector addition $\vec{PS} = \vec{PQ} + \vec{QS}$
 $\Rightarrow \vec{R} = \vec{A} + \vec{B}$

② Analytical Method \rightarrow



Let us extend \vec{A} in forward direction. Draw a perpendicular (\perp) ST on it. Let θ is the angle between \vec{A} & \vec{B} .

In the ΔPST $(PS)^2 = (PT)^2 + (ST)^2$
 $\Rightarrow R^2 = (PQ + QT)^2 + (ST)^2$
 $\Rightarrow R^2 = (PQ)^2 + (QT)^2 + 2 \cdot PQ \cdot QT + (ST)^2$
 $\Rightarrow R^2 = (PQ)^2 + (QT)^2 + (ST)^2 + 2 \cdot PQ \cdot QT$
 $\Rightarrow R^2 = A^2 + B^2 + 2 \cdot A \cdot B \cos \theta$ — (I)

In the ΔQST $\cos \theta = \frac{QT}{QS} = \frac{QT}{B}$
 $\Rightarrow QT = B \cos \theta$ — (II)

$\sin \theta = \frac{ST}{QS} = \frac{ST}{B}$
 $\Rightarrow ST = B \sin \theta$ — (III)

Using eqn (II) in eqn (I)

$R^2 = A^2 + B^2 + 2AB \cos \theta$
 $\Rightarrow R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$ — (IV)

This gives the magnitude of resultant vector.
 Let β be the angle between \vec{A} & \vec{R} in the ΔPST .

$$\tan \beta = \frac{ST}{PT} = \frac{ST}{PQ+QT}$$

$$= \frac{B \sin \theta}{A+B \cos \theta}$$

$$\Rightarrow \beta = \tan^{-1} \left(\frac{B \sin \theta}{A+B \cos \theta} \right) \quad \text{--- (v)}$$

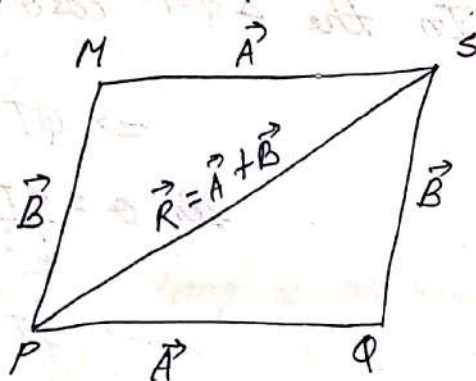
This gives the direction of \vec{R} .

(ii) Parallelogram law of vector addition

Statement \rightarrow If two vectors (\vec{A} & \vec{B}) are represented in magnitude & direction by the two adjacent sides of a parallelogram taken in opposite order draw from a point then the diagonal of the parallelogram drawn from the same point will represent the resultant vector in magnitude and direction.

It can also be proved by two methods \div

(a) Graphical method \rightarrow



In the fig: PQSM is the parallelogram in which.

$$\vec{PQ} = \vec{A}$$

$$\vec{PM} = \vec{B}$$

In a parallelogram the opposite sides are equal & parallel
 so $\vec{PM} = \vec{QS} = \vec{B}$.

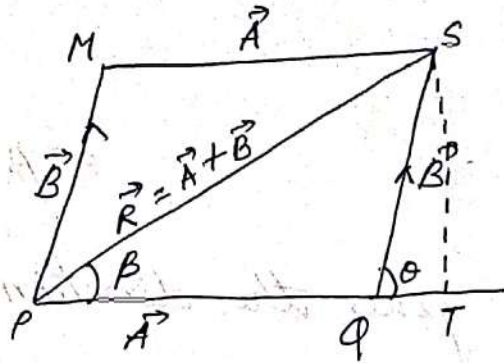
In the ΔPQS

$$\vec{PS} = \vec{PQ} + \vec{QS}$$

$$\Rightarrow \vec{R} = \vec{A} + \vec{B}$$

$$\Rightarrow \boxed{\vec{R} = \vec{A} + \vec{B}}$$

(b) Analytical Method \rightarrow



Let us extend \vec{A} in forward direction. Draw a \perp ST on it. Let θ is the angle between \vec{A} & \vec{B} .

$$\text{In the } \Delta PST \Rightarrow (PS)^2 = (PT)^2 + (ST)^2$$

$$\Rightarrow R^2 = (PQ + QT)^2 + (ST)^2$$

$$\Rightarrow R^2 = PQ^2 + QT^2 + 2 \cdot PQ \cdot QT + (ST)^2$$

$$\Rightarrow R^2 = PQ^2 + (QT)^2 + (ST)^2 + 2 \cdot PQ \cdot QT$$

$$\Rightarrow R^2 = A^2 + B^2 + 2 \cdot A \cdot QT \quad \text{--- (i)}$$

$$\text{In the } \Delta QST \quad \cos \theta = \frac{QT}{QS} = \frac{QT}{B}$$

$$\Rightarrow QT = B \cos \theta \quad \text{--- (ii)}$$

$$\sin \theta = \frac{ST}{QS} = \frac{ST}{B}$$

$$\Rightarrow ST = B \sin \theta \quad \text{--- (iii)}$$

Using eqn (ii) in eqn (i)

$$R^2 = A^2 + B^2 + 2AB \cos \theta$$

$$\Rightarrow R = \sqrt{A^2 + B^2 + 2AB \cos \theta} \quad \text{--- (iv)}$$

This gives the magnitude of resultant vector.

Let β be the angle between \vec{A} & \vec{R} in the $\triangle PST$

$$\tan \beta = \frac{ST}{PT} = \frac{ST}{PQ+QT}$$

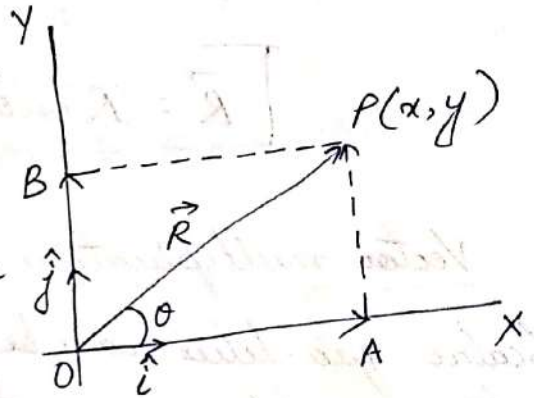
$$= \frac{B \sin \theta}{A+B \cos \theta}$$

$$\Rightarrow \beta = \tan^{-1} \left(\frac{B \sin \theta}{A+B \cos \theta} \right) \quad \text{--- (v)}$$

This gives the direction of \vec{R} .

Resolution of Vectors \rightarrow Resolution of vectors is the process of obtaining the component vectors which when combined, according to laws of vector addition, produce the given vector.

Rectangular components \rightarrow Rectangular components of a given vector are its components in two mutually perpendicular directions in the plane of the given vector.



Let $\vec{OP} = \vec{R}$ be the position vector of a point $P(x, y)$. From P draw PA and PB perpendiculars on x -axis & y -axis respectively.

Thus $OA = x$ & $OB = y$

If \hat{i} & \hat{j} are the unit vectors along x -axis and y -axis

$$\vec{OA} = x\hat{i} \quad \& \quad \vec{OB} = y\hat{j}$$

Applying triangle's law in $\triangle OAP$

$$\vec{OP} = \vec{OA} + \vec{AP} = \vec{OA} + \vec{OB}$$

$$\vec{R} = x\hat{i} + y\hat{j}$$

$\vec{OA} = x\hat{i}$ & $\vec{OB} = y\hat{j}$ are called x -component & y -component of \vec{R} respectively. Let θ be the angle which the given vector \vec{R} makes with x -axis.

$$\begin{aligned} \text{In } \triangle OAP \\ \cos \theta = \frac{OA}{OP} &\Rightarrow \frac{x}{R} = \cos \theta \\ &\Rightarrow x = R \cos \theta \end{aligned}$$

$$\begin{aligned} \text{In } \triangle OAP \\ \sin \theta = \frac{AP}{OP} \\ \Rightarrow \sin \theta = \frac{OB}{OP} \\ \Rightarrow y = R \sin \theta \end{aligned}$$

$$\boxed{\vec{R} = (R \cos \theta) \hat{i} + (R \sin \theta) \hat{j}}$$

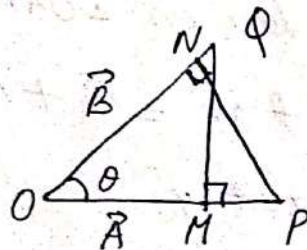
Vector multiplication

Scalar quantities can be multiplied by simple algebra but vector quantities are not multiplied simply by algebra.

There are two types :- (i) Dot product
(ii) Cross product

1) Dot product \rightarrow Dot product of two vectors is defined as the product of their magnitudes and smaller cosine angle between them i.e; $\vec{A} \cdot \vec{B} = AB \cos \theta$ - (1)

when θ is the smaller angle between \vec{A} & \vec{B}



$$\text{In the } \triangle OQM \quad \cos \theta = \frac{OM}{OQ} = \frac{OM}{B}$$

$$\rightarrow OM = B \cos \theta$$

Using eqn (ii) in eqn (i)

$$\vec{A} \cdot \vec{B} = A(ON)$$

$$= A(\text{component of } \vec{B} \text{ along } \vec{A}) \quad \text{--- (iii)}$$

In the $\triangle OPN$

$$\cos \theta = \frac{ON}{OP}$$

$$\Rightarrow ON = A \cos \theta \quad \text{--- (iv)}$$

Using eqn (iv) in eqn (i)

$$\vec{A} \cdot \vec{B} = B(ON)$$

$$= B(\text{component of } \vec{A} \text{ along } \vec{B}) \quad \text{--- (v)}$$

From eqn (iii) & (v) the dot product of two vectors may be alternatively defined as the product of the magnitude of one vector with the component of the other vector in the direction of first vector.

Properties of dot product

(i) Its commutative i.e; $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

(ii) Its distributive i.e; $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$

Special cases

Case-I For equal vectors ($\theta = 0^\circ$)

$$\vec{A} \cdot \vec{A} = AA(\cos 0^\circ) \\ = A^2$$

Therefore the dot product of two equal vectors is equal to the square of its magnitude.

$$\text{Hence, } \hat{i} \cdot \hat{i} = 1$$

$$\hat{j} \cdot \hat{j} = 1$$

$$\hat{k} \cdot \hat{k} = 1$$

Case-II On parallel vectors ($\theta = 0$)

$$\vec{A} \cdot \vec{B} = AB(\cos 0) \\ = AB$$

Therefore the dot product of two parallel vectors is equal to the product of their magnitudes.

Case-III For anti-parallel vectors ($\theta = 180^\circ$)

$$\vec{A} \cdot \vec{B} = AB \cos 180^\circ \\ = -AB$$

Therefore the dot product of two anti-parallel vectors is also equal to the product of their magnitudes but it is negative.

Case-IV For perpendicular vectors ($\theta = 90^\circ$)

$$\vec{A} \cdot \vec{B} = AB \cos 90^\circ \\ = 0$$

Therefore the dot product of two perpendicular vectors always 0. This is also called the condition of perpendicular

$$\text{Hence, } \hat{i} \cdot \hat{j} = 0 \quad \hat{k} \cdot \hat{i} = 0 \\ \hat{j} \cdot \hat{i} = 0 \quad \hat{i} \cdot \hat{k} = 0 \\ \hat{j} \cdot \hat{k} = 0 \\ \hat{k} \cdot \hat{j} = 0$$

(2) Cross product \rightarrow Cross product of two vectors \vec{A} & \vec{B} is a vector quantity (\vec{R}) whose magnitude is equal to the product of their magnitudes and smaller sine angle between them. The direction of \vec{R} is perpendicular to the plane containing \vec{A} & \vec{B} i.e; $\vec{A} \times \vec{B} = \vec{R} = AB \sin \theta (\hat{n})$

\hat{n} is the unit vector perpendicular to the plane containing \vec{A} & \vec{B} .

The magnitude of \vec{R} is given by $|\vec{R}| = AB \sin \theta$

The direction can be obtained by different rules are

(i) Right hand thumb rule \rightarrow Imagine the thumb of your right hand along a direction perpendicular to the plane containing \vec{A} & \vec{B} . And the other fingers curl from \vec{A} to \vec{B} through the smaller angle between them. Then the direction of the thumb gives the direction of \vec{R} . $\vec{A} \times \vec{B} = \vec{R}$

(ii) Right hand's screw rule \rightarrow Imagine a right hand screw placed perpendicular to the plane containing \vec{A} & \vec{B} rotate the cap of the screw from \vec{A} to \vec{B} through the smaller angle. The direction of the motion of the tip of the screw gives the direction of \vec{R} . $\vec{A} \times \vec{B} = \vec{R}$

Properties of cross product

(i) It is non-commutative $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$, but $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

(ii) It is distributive $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$.

Special cases

Case-I For equal vectors $\theta = 0^\circ$

$$\begin{aligned}\vec{A} \times \vec{B} &= AB \sin 0 (\hat{n}) \\ &= 0(\hat{n}) \\ &= \text{Null vector}\end{aligned}$$

$$|\vec{A} \times \vec{B}| = 0$$

Therefore,

$$\begin{aligned}|\hat{i} \times \hat{i}| &= 0 \\ |\hat{j} \times \hat{j}| &= 0 \\ |\hat{k} \times \hat{k}| &= 0\end{aligned}$$

Case-II For parallel vectors ($\theta = 0^\circ$)

$$\begin{aligned}\vec{A} \times \vec{B} &= AB \sin 0 (\hat{n}) \\ &= 0(\hat{n}) \\ &= \text{Null vector}\end{aligned}$$

$$|\vec{A} \times \vec{A}| = 0$$

Case-III For anti-parallel vectors ($\theta = 180^\circ$)

$$\begin{aligned}\vec{A} \times \vec{B} &= AB \sin 180^\circ (\hat{n}) \\ &= 0(\hat{n}) \\ &= \text{Null vector.}\end{aligned}$$

$$|\vec{A} \times \vec{A}| = 0$$

Case-IV For perpendicular vectors ($\theta = 90^\circ$)

$$\begin{aligned}\vec{A} \times \vec{B} &= AB \sin 90^\circ (\hat{n}) \\ &= AB(\hat{n})\end{aligned}$$

$$|\vec{A} \times \vec{B}| = AB$$

$$\begin{aligned}\therefore \hat{i} \times \hat{j} &= \hat{k} & \hat{j} \times \hat{k} &= \hat{i} & \hat{k} \times \hat{i} &= \hat{j} \\ \hat{j} \times \hat{i} &= -\hat{k} & \hat{k} \times \hat{j} &= -\hat{i} & \hat{i} \times \hat{k} &= -\hat{j}\end{aligned}$$

UNIT-3 Kinematics

The branch of physics which deals with motion without considering the cause of motion is called kinematics.

Rest \rightarrow A body is said to be in rest if it doesn't change its position with respect to the surrounding.

Motion \rightarrow A body is said to be in motion if it changes its position with respect to its surrounding.

Rested motion are not absolute. They are relative. A body at rest with respect to one surrounding may be in motion with respect to another surrounding.

Ex \rightarrow A passenger sitting in a bus and the bus is moving, the passenger is at rest within respect to the bus but he is in motion with respect to the outside.

Terms connected with kinematics

① Distance \rightarrow The total path covered by a body is called distance. It is a scalar quantity.

Unit \rightarrow In C.G.S = cm
S.I = m

Dimensional Formula $[M^0 L^1 T^0]$

② Displacement \rightarrow The shortest distance between the two points is called displacement. It is obtained by joining the initial and final points by a straight line. It is always directed from initial to final.

Absolute rest or motion \rightarrow A body is said to be in absolute rest or motion if the reference point is taken to be one which is at rest. It is a vector quantity.

Unit \rightarrow C.G.S = cm
S.I = m

Dimension = $[M^0 L^1 T^0]$

(3) Speed \rightarrow The distance covered by a body in one second is called its speed.

$$\text{speed} = \frac{\text{Distance}}{\text{Time}}$$

It is a scalar quantity

$$\text{Unit} \rightarrow \text{C.G.S} = \text{cm/s}$$

$$\text{S.I} = \text{m/s}$$

$$\text{Dimension} \rightarrow [M^0 L^1 T^{-1}]$$

(4) Velocity \rightarrow The rate of change of displacement of a body is called its velocity. i.e; $\text{velocity} = \frac{\text{change in displacement}}{\text{time}}$

If \vec{s}_1 & \vec{s}_2 be the displacements of a body at times t_1 & t_2 respectively then the average velocity of the body is given by

$$\vec{v} = \frac{\vec{s}_2 - \vec{s}_1}{t_2 - t_1} \Rightarrow \vec{v} = \frac{\Delta \vec{s}}{\Delta t}$$

If Δt tends to zero then the average velocity becomes instantaneous velocity and is given by

$$\vec{v} = \frac{d\vec{s}}{dt}$$

$$\Rightarrow \vec{v} = \frac{d\vec{s}}{dt} \text{ (Instantaneous velocity)}$$

Velocity is a vector quantity.

There are two types of velocities.

1) Uniform velocity \rightarrow The velocity of a body is said to be uniform if it covers equal displacements in equal interval of time.

2) Non-uniform velocity \rightarrow The velocity of a body is said to be non uniform if it covers unequal displacements in equal intervals of time.

Unit \rightarrow C.G.S = cm/s

S.I = m/s

Dimensional Formula $\frac{[M^0 L^1 T^0]}{[T]} = [M^0 L^1 T^{-1}]$

Acceleration (\vec{a}) \rightarrow The rate of change of velocity of a body is called its acceleration.

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

$$\Rightarrow \vec{a} = \frac{\Delta \vec{v}}{t} \text{ (average acceleration)}$$

If $\Delta t \rightarrow 0$, then average acceleration becomes instantaneous acceleration. $\vec{a} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t}$

It is a vector quantity

There are two types of acceleration

1) Uniform acceleration \rightarrow The acceleration of a body is said to be uniform if it changes its velocity by equal amount in equal interval of time.

2) Non-uniform acceleration \rightarrow The acceleration of a body is said to be non uniform if it changes its velocity by unequal amounts in equal interval of time.

Units \rightarrow C.G.S = cm/s²

S.I = m/s²

Dimensional Formula $\rightarrow [M^0 L^1 T^{-2}]$

Acceleration in terms of displacement

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\vec{v} = \frac{d\vec{s}}{dt}$$

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\Rightarrow \vec{a} = \frac{d^2\vec{s}}{dt^2}$$

When a body moves with uniform velocity acceleration is zero. $\Rightarrow \left(\frac{d}{dt}\right)\left(\frac{d\vec{s}}{dt}\right)$

$$\Rightarrow \frac{d^2\vec{s}}{dt^2} = \vec{a}$$

Force \rightarrow Force is the basic cause of motion. It is an agent which creates or tends to create, destroys or tends to destroy, increases or decreases the motion in a body.

Force = mass \times acceleration

$$\boxed{F = ma}$$

(a) In S.I - Newton

$$\text{If } m = 1\text{kg}, a = 1\text{m/sec}^2$$

$$\text{Then } F = 1\text{N}$$

$$\text{i.e.; } \boxed{1\text{N} = 1\text{kg} \times 1\text{m/sec}^2}$$

\therefore 1N force is the amount of force which produces an acceleration of 1m/sec^2 in a body of mass 1kg.

(b) In C.G.S - Dyne

$$\text{If } m = 1\text{g}, a = 1\text{cm/sec}^2$$

$$\text{Then } F = 1\text{dyne} \quad \text{i.e.; } 1\text{dyne} = 1\text{gm} \times 1\text{cm/sec}^2$$

\therefore 1dyne force is the amount of force which produces an acceleration of 1cm/sec^2 in a body of mass 1gm.

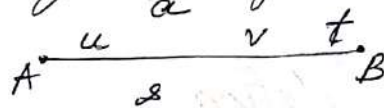
Relation between Newton & dyne

$$1N = 1kg \times 1m/sec^2$$
$$= 10^3 gm \times 10^2 cm/sec^2$$

$$1N = 10^5 \text{ dyne}$$

Equation of motion under gravity

(i) Equation in one dimensional motion \rightarrow Consider a body moving with uniform acceleration (a) from A to B. The initial velocity of the body at A is ' u ' and the final velocity at B is ' v '. Let the time taken by the body to travel from A to B is ' t ' second. The displacement from A to B is ' s '. The relation or equation connecting u, v, s, a and t is known as equation of kinematics or one dimensional equation.



① Velocity after t sec ($\vec{v} = \vec{u} + \vec{a}t$)

Pf: we know, $\vec{a} = \frac{d\vec{v}}{dt}$

$$\Rightarrow d\vec{v} = \vec{a} dt$$

Integrating both sides with the limits, we have.

$$\int_u^v d\vec{v} = \int_0^t \vec{a} dt$$

$$\Rightarrow [\vec{v}]_u^v = \vec{a} [t]_0^t$$

$$\Rightarrow \vec{v} - \vec{u} = \vec{a} [t - 0]$$

$$\Rightarrow \vec{v} - \vec{u} = \vec{a} t$$

$$\Rightarrow \boxed{\vec{v} = \vec{u} + \vec{a} t}$$

- (2) Displacement after 't' second $s = ut + \frac{1}{2}at$
- (3) Velocity after a displacement $v^2 = u^2 + 2as$
- (4) Displacement in n^{th} sec. $S_n = u + \frac{a}{2}(2n-1)$

Equation under gravity \rightarrow
 Downward motion

- (a) Velocity-time relation $v = u + gt$
- (b) Displacement-time relation $x = ut + \frac{1}{2}gt^2$
- (c) Velocity displacement relation $v^2 - u^2 = 2gx$
- (d) Displacement in n^{th} second $S_n^{\text{th}} = u + \frac{g}{2}(2n-1)$

Upward motion

- (i) $v = u - gt$
- (ii) $x = ut - \frac{1}{2}gt^2$
- (iii) $v^2 = u^2 - 2gx$
- (iv) $S_n^{\text{th}} = u - \frac{g}{2}(2n-1)$

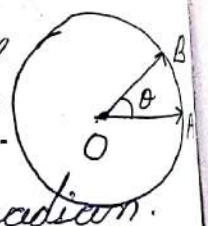
If instant

Circular Motion

The motion of a body is said to be circular if it moves in such a way that its distance from a particular point always remains fixed. The fixed point is called centre of the circle and the fixed

Terms connected with circular motion

(1) Angular displacement (θ) \rightarrow The angle turned by a radius vector is called angular displacement. It is denoted by a unit. It is measured in radian.



Relation between angular displacement & linear displacement

In the arc OAB $\theta = \frac{AB}{OA} = \frac{s}{r}$

$\Rightarrow s = r\theta$

\therefore i.e; linear displacement = radius \times angular displacement.

② Angular velocity (ω) \rightarrow The rate of change of angular displacement is called angular velocity. If θ_1 & θ_2 be the angular displacements in time t_1 & t_2 respectively, then the angular velocity $\omega = \frac{\theta_2 - \theta_1}{t_2 - t_1}$

$$\omega = \frac{\Delta\theta}{\Delta t}$$

If $\Delta t \rightarrow 0$ then the average angular velocity becomes instantaneous angular velocity and is given by

$$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t}$$

$$\omega = \frac{d\theta}{dt}$$

Unit \rightarrow radian/second

Relation between linear velocity and angular velocity

$$\begin{aligned} \text{we know, } v &= \frac{ds}{dt} \\ &= \frac{d(r\theta)}{dt} \\ &= r \frac{d\theta}{dt} \end{aligned}$$

$$\Rightarrow v = r\omega$$

i.e; linear velocity = radius \times angular velocity.

③ Angular acceleration (α) \rightarrow The rate of change of angular velocity is called angular acceleration. If ω_1 & ω_2 be the angular velocities at times t_1 & t_2 respectively then the average angular acceleration of the body is given by $\alpha = \frac{\omega_2 - \omega_1}{t_2 - t_1}$

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

If $\Delta t \rightarrow 0$ then the average angular acceleration becomes instantaneous angular acceleration and is given by

$$\alpha = \lim_{\Delta t \rightarrow 0} \frac{\Delta \omega}{\Delta t}$$

$$\alpha = \frac{d\omega}{dt}$$

Unit \rightarrow radian/sec²

Relation between linear acceleration & angular acceleration

we know, $a = \frac{dv}{dt}$

$$= \frac{d(r\omega)}{dt}$$

$$= r \frac{d\omega}{dt}$$

$$\Rightarrow \boxed{a = r\alpha}$$

i.e; linear acceleration = radian \times angular acceleration

In vector form

1) Relation between linear displacement & angular displacement $\vec{s} = \vec{r} \times \vec{\theta}$.

2) Relation between linear velocity & angular velocity $\vec{v} = \vec{r} \times \vec{\omega}$.

3) Relation between linear acceleration & angular acceleration

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$= \frac{d}{dt} (\vec{r} \times \vec{\omega})$$

$$= r \times \frac{d\vec{\omega}}{dt} + \frac{d\vec{r}}{dt} \times \vec{\omega}$$

$$\Rightarrow \boxed{\vec{a} = \vec{r} \times \vec{\alpha} + \vec{v} \times \vec{\omega}}$$

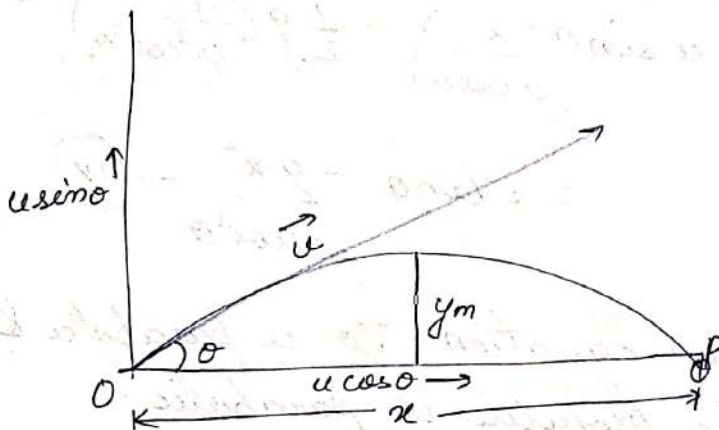
Projectile Motion \rightarrow Whenever a body is thrown into the space and it is not influenced by any external force is called projectile.

Eg \rightarrow (i) A bomb dropped from an aeroplane.

(ii) A bullet fired from a rifle.

(iii) A bag dropped from a running bus.

Projectile fired at an $\angle \theta$ with the horizontal range \rightarrow



Consider a projectile projected at an angle θ with the horizontal direction with a velocity ' u '. The velocity ' u ' has two components

(i) $u \cos \theta$ acting horizontally which is uniform throughout the motion.

(ii) $u \sin \theta$ along vertically upward direction which is non uniform and decreases due to gravity and becomes zero at the highest point.

The following points are to be noted \rightarrow

(i) Eqn. of trajectory \rightarrow It is an eqn. connecting the x -displacement & the y -displacement of the projectile.

Applying the eqn. of kinematics

$$s = ut + \frac{1}{2}at^2$$

$$x = u \cos \theta t + \frac{1}{2} 0 t^2$$

$$x = u \cos \theta t \quad \text{--- (i)}$$

$$\text{Along } y\text{-axis} \rightarrow y = u \sin \theta t + \frac{1}{2} (-g) t^2 \\ = u \sin \theta t - \frac{1}{2} g t^2 \quad \text{--- (ii)}$$

$$\text{From eqn (i)} \quad t = \frac{x}{u \cos \theta} \quad \text{--- (iii)}$$

Using eqn (iii) in eqn (ii)

$$y = u \sin \theta \left(\frac{x}{u \cos \theta} \right) - \frac{1}{2} g \left(\frac{x^2}{u^2 \cos^2 \theta} \right) \\ = x \tan \theta - \frac{g x^2}{2 u^2 \cos^2 \theta} \quad \text{--- (iv)}$$

This is an equation to a parabola & hence the path of the projectile is parabolic.

(ii) Maximum height (y_m) \rightarrow It is the maximum distance covered by the projectile in the vertical direction.

Considering the motion in the vertical direction.

The initial velocity at O = $u \sin \theta$

final velocity at P = 0

acceleration = $-g$

displacement = y_m

Applying the eqn. of kinematics

$$V^2 - U^2 = 2as$$

$$\text{we have, } 0^2 - u^2 \sin^2 \theta = 2(-g)y_m$$

$$\Rightarrow y_m = \frac{u^2 \sin^2 \theta}{2g} \quad \text{--- (v)}$$

(iii) Time of ascent (t) \rightarrow It is the time taken by the projectile to reach the highest point.

Consider the motion of the projectile in the vertically upward direction.

The initial velocity = $u \sin \theta$

Final velocity = 0

acceleration = $-g$

time taken = t

Applying the eqn. of kinematics

$$v = u + at$$

$$0 = u \sin \theta + (-g)t$$
$$= u \sin \theta - gt$$

$$gt = u \sin \theta$$

$$t = \frac{u \sin \theta}{g} \quad \text{--- (VI)}$$

(iv) Time of descent (t') \rightarrow It is the time taken by the projectile to come down from the highest point to the level of projection considering the motion in vertically downward direction.

The initial velocity = 0

acceleration = g

displacement = y_m

time taken = t'

Applying the eqn. of kinematics

$$s = ut + \frac{1}{2} at^2$$

$$\text{we have } y_m = 0 \cdot t' + \frac{1}{2} g t'^2$$

$$= t' = \frac{2y_m}{g} = \frac{2}{g} \cdot \frac{u^2 \sin^2 \theta}{2g} = \frac{u^2 \sin^2 \theta}{g}$$

$$t' = \frac{u \sin \theta}{g} \quad \text{--- (vii)}$$

Since the -ve sign has no meaning
 $\therefore t' = \frac{u \sin \theta}{g}$ --- (viii)

(v) Total time of flight (T) \rightarrow It is the sum of the time of ascent & the time of descent

$$T = t + t'$$
$$= \frac{u \sin \theta}{g} + \frac{u \sin \theta}{g}$$

$$T = \frac{2u \sin \theta}{g} \quad \text{--- (ix)}$$

(vi) Horizontal range (x) \rightarrow It is the distance covered by the projectile in the horizontal direction.

It is given by $x = \text{horizontal velocity} \times \text{total time of flight}$

$$= u \cos \theta \times \frac{2u \sin \theta}{g}$$

$$\Rightarrow x = \frac{u^2 \sin^2 \theta}{g} \quad \text{--- (x)}$$

Condition for maximum horizontal range \rightarrow The horizontal range of a projectile projected at an angle θ with the horizontal direction is given by

$$x = \frac{u^2 \sin^2 \theta}{g}$$

The range will be maximum if

$$\sin 2\theta = 1 = \sin 90^\circ$$

$$\Rightarrow 2\theta = 90^\circ$$

$$\Rightarrow \theta = 45^\circ$$

∴ A projectile covers maximum range when it is projected at an angle of 45° with the horizontal. The maximum range is given by

$$X_{\max} = \frac{u^2}{g}$$

Two angle of projections for the same range \rightarrow The horizontal range of a projectile projected at an angle θ is given by

$$R = \frac{u^2 \sin^2 \theta}{g} \quad \text{--- (1)}$$

$$\Rightarrow R = \frac{u^2 \sin^2 (180^\circ - 2\theta)}{g}$$

$$\Rightarrow R = \frac{u^2 \sin^2 (90^\circ - \theta)}{g} \quad \text{--- (2)}$$

From eqn (1) & (2) it is clear that there are two angle of projection for the same range. If one is θ the other is $(90^\circ - \theta)$.

Work \rightarrow Work is said to be done if a body displaces from one position to another position for the application of force.

$W = \vec{F} \cdot \vec{S}$ i.e.; the dot product of force & displacement.

From the definition of dot product $W = FS \cos \theta$
 $= F(S \cos \theta)$
 $= S(F \cos \theta)$

where θ is the smaller angle between the force & the displacement. The work done may also be defined as the product of the magnitude of the force and the component of the displacement along the direction of the force or it may also be defined as the magnitude of the displacement and the component of the force along the direction of displacement.

Since, work is the dot product of two vectors so it is a scalar quantity.

S.I unit of work \rightarrow There are two types of unit in which work is measured.

(i) Absolute unit

(ii) Gravitational unit.

(i) Absolute unit \rightarrow

(a) C.G.S system \rightarrow Eg If $F = 1 \text{ dyne}$, $s = 1 \text{ cm}$, $\theta = 0^\circ$

$$\begin{aligned} W &= F \cdot S \cos \theta \\ &= 1 \times 1 \times 1 \\ &= 1 \text{ erg} \end{aligned}$$

The work done is said to be 1 erg if a force of 1 dyne displaces the body through a distance of 1 cm in the direction of force.

(b) S.I \rightarrow Joule

If $F = 1 \text{ Newton}$, $s = 1 \text{ m}$, $\theta = 0^\circ$

$$W = 1 \times 1 \times 1 = 1 \text{ Joule}$$

\therefore Work done is said to be 1 Joule.

If a force of 1 Newton displaces a body through 1 m in the direction of force.

(ii) Gravitational unit \rightarrow

(a) In C.G.S system \rightarrow gm cm

If $F = 1 \text{ gm wt}$ or 1 gf , $s = 1 \text{ cm}$, $\theta = 0^\circ$

$$W = 1 \text{ gm wt} \times 1 \text{ cm} \times 1 \\ = 1 \text{ gm cm}$$

The work done is said to be 1 gm cm if a force of 1 gm wt displaces a body through 1 cm in the direction of force.

(b) S.I system \rightarrow kg m

If $F = 1 \text{ kg wt}$ / 1 kg f , $s = 1 \text{ m}$, $\theta = 0^\circ$

$$W = 1 \times 1 \times 1 \\ = 1 \text{ kg m}$$

The work done is said to be 1 kg m if a force of 1 kg wt displaces a body through 1 m in the direction of force.

Dimensional Formula \rightarrow

$$F = [MLT^{-2}][L]$$

$$= [ML^2T^{-2}]$$

Friction \rightarrow Whenever a body tends to slide over a surface, an opposing force acts tangentially at the interface called as force of friction.

There are three types of friction

- (i) Sliding friction
- (ii) Rolling friction
- (iii) Fluid friction

(i) Sliding friction \rightarrow The force of friction which comes into play between two surfaces when one tends to slide over the other is called sliding friction.

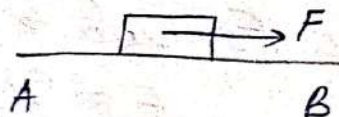
(a) Static friction \rightarrow The static friction is the force of friction between two surfaces so long as there is no relative motion between them.

(b) Limiting friction \rightarrow It is the maximum value of force of friction between two surfaces so long as there is no relative motion between them.

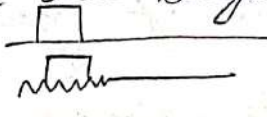
(c) Dynamic friction \rightarrow It is the force of friction which comes into play between two surfaces when there is some relative motion between them.

Laws of limiting friction

- 1) The force of friction always acts opposite to the direction of motion.



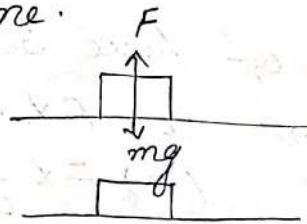
2) The force of limiting friction depends upon the nature of the surface in contact and acts tangentially to the interface between the two surfaces.



3) The magnitude of the limiting friction is directly proportional to the magnitude of the normal reaction.
i.e; $F \propto R$



4) The magnitude of the limiting friction is independent of the area of contact so long as the normal reaction remains the same.



Co-efficient of friction (μ) \rightarrow It is defined as the ratio between the force of limiting friction to the normal reaction i.e; $\mu = \frac{F}{R}$

Since, the force of limiting friction depends upon the nature of the surface in contact so the co-efficient of friction also depends upon the nature of the surface in contact.

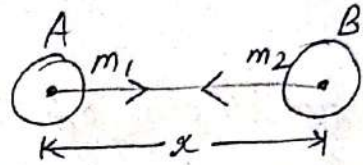
Methods of reducing the friction

- 1) By converting sliding friction into rolling friction.
- 2) By use of lubricants at the interface, friction can be reduced.
- 3) By using ball bearing arrangement.
- 4) By stream lining, friction can be reduced.

UNIT-5 Gravitation

Newton's law of gravitation \rightarrow It states that every body in the universe attracts every other body with force whose magnitude is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

Explanation \rightarrow Consider two bodies A & B having masses m_1 & m_2 separated by a distance (x).



If 'F' be the magnitude of force between them according to the statement

(i) $F \propto m_1 m_2$
(ii) $F \propto \frac{1}{x^2}$

Combiningly both

$$F \propto \frac{m_1 m_2}{x^2}$$

$$\Rightarrow F = G \frac{m_1 m_2}{x^2} \text{ where } G = \text{universal gravitational force}$$

If $m_1 = m_2 = 1 \text{ unit}$ if $x = 1 \text{ unit}$

$$\text{Then, } \boxed{F = G}$$

Therefore, universal gravitational constant may be defined as the force of attraction between the two bodies each of unit mass separated by a unit distance.

Unit of Gravitation (G)

$$\text{From eqn (1) } G = \frac{F x^2}{m_1 m_2}$$

So, its unit is

$$\text{C.G.S.} \rightarrow \text{dyne cm}^2 / \text{gm}^2$$

$$\text{I.T.S.} \rightarrow \text{N m}^2 / \text{kg}^2$$

Dimensional formulae $\rightarrow G = \frac{F r^2}{m_1 m_2}$

$$= \frac{[M L T^{-2}] [L^2]}{[M][M]}$$

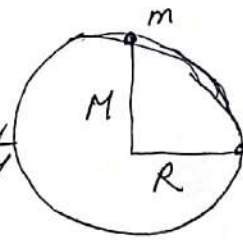
$$= [M^{-1} L^3 T^{-2}]$$

Value of G \rightarrow In C.G.S $\rightarrow 6.67 \times 10^{-8}$ dyne cm^2/gm^2
 In S.I $\rightarrow 6.67 \times 10^{-11}$ Newton m^2/kg^2

Acceleration due to gravity (g) \rightarrow The force with which earth attracts a body towards its centre is called gravity.

The acceleration produced due to gravity is called acceleration due to gravity.

Consider a body of mass 'm' placed on the surface of earth the force of gravity on the body is equal to the weight i.e;



$$F = W = mg \quad \text{--- (i)}$$

According to Newton's law of gravitation the force between the earth and the body is given by

$$F = \frac{G M m}{R^2} \quad \text{--- (ii)}$$

$$\text{Comparing } mg = \frac{G M m}{R^2}$$

$$\Rightarrow g = \frac{G M}{R^2} \quad \text{--- (iii)}$$

This is the expression for acceleration due to gravity here 'G' is the gravitational constant.

M & R are mass & radius of the earth.

Units (g) C.G.S = cm/sec^2
S.I = m/sec^2

Value of 'g' on the surface of earth C.G.S $\rightarrow 980 \text{ cm/sec}^2$
S.I $\rightarrow 9.8 \text{ m/sec}^2$

Mass of earth

We know, $g = \frac{GM}{R^2}$

$$M = \frac{gR^2}{G}$$

$$= \frac{9.8 \times (6.38 \times 10^6)^2}{6.67 \times 10^{-11}} = 5.98 \times 10^{24} \text{ kg}$$

Density of earth

$$g = \frac{GM}{R^2}$$

$$= \frac{G \times V \times \rho}{R^2} = \frac{G \times \frac{4}{3} \pi R^3 \times \rho}{R^2}$$

$$= G \times \frac{4}{3} \pi R \times \rho$$

$$\Rightarrow \rho = \frac{3g}{4\pi R G}$$

$$= \frac{3 \times 9.8}{4 \times 3.14 \times 6.38 \times 10^6 \times 6.67 \times 10^{-11}}$$

$$= 5497 \text{ kg/m}^3$$

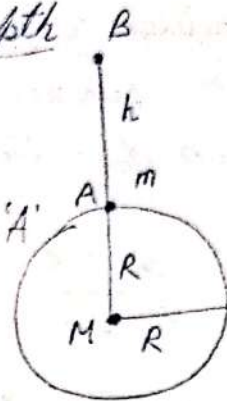
Mass \rightarrow The amount of matter contained in a body.
Weight \rightarrow The amount of matter by which the earth attracts the body by its gravitational pull.

Variation of 'g' with altitude and depth

Altitude \rightarrow Consider a body of mass 'm' placed on the surface of earth. The acceleration due to gravity at a point 'A' on the surface of earth is given by

$$g = \frac{GM}{R^2} \quad \text{--- (i)}$$

where M & R are mass & radius of earth.



If the body is taken to a height 'h' from the surface of earth then the acceleration due to gravity at that point B is given by $g' = \frac{GM}{(R+h)^2}$ --- (ii)

Dividing eqn (ii) by eqn (i)

$$\frac{g'}{g} = \frac{R^2}{(R+h)^2} = \frac{R^2}{R^2 \left(1 + \frac{h}{R}\right)^2}$$

$$= \left(1 + \frac{h}{R}\right)^{-2}$$

$$= 1 - \frac{2h}{R} \quad \left(\text{Expanding Binomially \& neglecting the higher order terms.}\right)$$

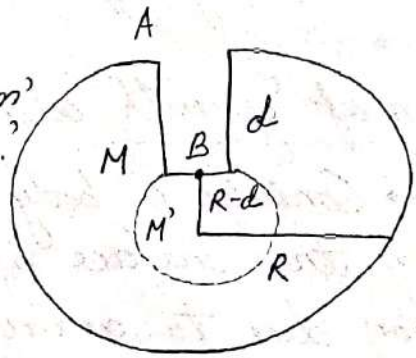
$$\Rightarrow g' = \left(1 - \frac{2h}{R}\right)g$$

$$\Rightarrow g' = g - \frac{2hg}{R}$$

$$\Rightarrow g - g' = \frac{2hg}{R} \quad \text{--- (iii)}$$

i.e; the change in acceleration due to gravity is directly proportional to height, thus it is clear that the acceleration due to gravity decreases with an increase in height.

Depth \rightarrow Consider a body of mass 'm' placed on the surface of earth at 'A'. The acceleration due to gravity at the point 'A'.



$$g = \frac{GM}{R^2}$$

$$= \frac{G \times V \times \rho}{R^2} = \frac{G \times \frac{4}{3} \pi R^3 \rho}{R^2}$$

$$\boxed{g = \frac{4}{3} \pi R \rho G} \quad \text{--- (1)}$$

Let the body be taken to a point B at a depth of 'd' from the surface of earth. The acceleration due to gravity at the point B.

$$g' = \frac{GM'}{(R-d)^2} \quad (M' \text{ is the mass of the inner sphere of radius } (R-d))$$

$$= \frac{G \times \frac{4}{3} \pi (R-d)^3 \rho}{(R-d)^2}$$

$$\Rightarrow g' = \frac{4}{3} \pi (R-d) \rho G \quad \text{--- (2)}$$

Dividing eqn (2) by eqn (1)

$$\frac{g'}{g} = \frac{R-d}{R}$$

$$= 1 - \frac{d}{R} \Rightarrow g' = g - \frac{dg}{R}$$

$$\Rightarrow g - g' = \frac{dg}{R} \quad \text{--- (iii)}$$

Since, g & R are constants so the change in acceleration due to gravity is proportional to the depth d . Thus the acceleration due to gravity decreases with depth from the surface of earth.

At the centre of earth $d=R$, therefore $g'=0$

The value of acceleration due to gravity at the centre of earth is 0. So the weight of the body centre of earth is 0.

Kepler's laws of Planetary Motion → Kepler gave three laws on planetary motion.

(i) Kepler's first law/Law of elliptical orbit → It states that a planet revolves around the sun in an elliptical path and the sun is situated at one of its foci.

(ii) Kepler's second law/Law of areal velocity → It states that a planet moves around the sun in such a way that its areal velocity is always constant i.e.; its radius vector sweeps equal area in equal interval of time.

(iii) Kepler's third law/Law of time period → It states that a planet moves around the sun in a such a way that the square of its time period is proportional to cube of its semi major axis i.e.; $T^2 \propto R^3$.

Simple harmonic Motion \rightarrow S.H.M is the motion in which the restoring force is proportional to displacement from the mean position and opposes its increase.

Units

$$\text{C.G.S.} = \text{dyne m}^{-1}$$

$$\text{S.I. \& M.K.S} = \text{N m}^{-1}$$

Any system that repeats its motion to and fro its mean or rest point executes simple harmonic motion.

Ex - simple pendulum, mass spring system.

a steel ruler clamped to a bench oscillates when its free end is displaced sideways.

a steel ball rolling in a curved dish
a swing.

Thus to get S.H.M a body is displaced away from its rest position and then released. The body oscillates due to restoring force. Under the action of this restoring force the body accelerates and overshoots the rest position due to inertia. The restoring force then pulls it back.

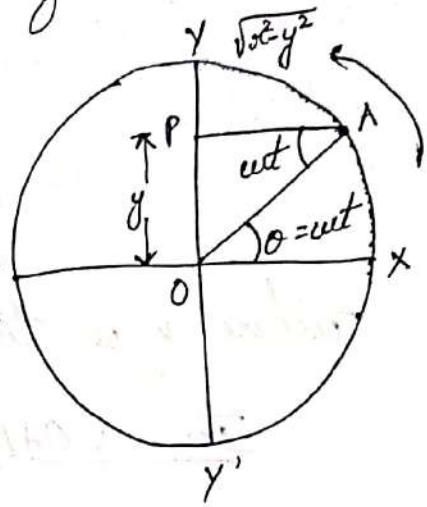
The restoring force is always directed towards the mean position and so the acceleration is also directed towards the mean or rest position.

Characteristics of S.H.M

(i) Displacement (y) - Amplitude (x)

Displacement of a particle vibrating in S.H.M at any instant, it is defined as its distance from the mean position at that instant.

Let 'P' be the position of projection of A, at any instant, of time 't'. Time 't' is measured from the instant when P was at O.



In ΔOAP , $\frac{OP}{AO} = \sin \theta$

$$OP = OA \sin \theta$$

$$y = x \sin \omega t \quad \text{--- (1)} \quad \left(\omega = \frac{\theta}{t} \text{ or } \theta = \omega t \right)$$

where 'x' is the radius of circle and 'omega' is the angular velocity of A or P.

It is clear from eqn (1) that 'y' changes with time, 'y' is maximum when $\sin \omega t$ is maximum.

Since extreme values of $\sin \omega t = \pm 1$

$$y = \pm x$$

'x' is called the amplitude of vibrations.

Amplitude of a particle, vibrating in S.H.M., is defined as its maximum displacement on either side of the mean position.

Unit of 'y' and 'x' are cm or metre.

(ii) Velocity \rightarrow We get velocity 'V' of the point P by differentiating eqn (i) w.r.t. time.

$$V = \frac{dy}{dt} = \frac{d}{dt} (x \sin \omega t)$$

$$= x \cos \omega t \frac{d}{dt} (\omega t)$$

$$= x \omega \cos \omega t$$

$$V = v \cos \omega t \quad \text{--- (ii)}$$

where v is the linear velocity of particle A in m/s

In $\triangle OAP$,

$$\cos \omega t = \frac{AP}{OA} = \frac{\sqrt{x^2 - y^2}}{x}$$

$$V = \frac{v \sqrt{x^2 - y^2}}{x} \quad \text{--- (iii)}$$

$$(v = x\omega)$$

$$V = x\omega \frac{\sqrt{x^2 - y^2}}{x} = \omega \sqrt{x^2 - y^2} \quad \text{--- (iv)}$$

Eqn (ii) gives the velocity of P at various instants of time while eqn (iii) and (iv) gives its velocity at different displacements.

At 0, $y = 0$

$$\text{From eqn (iv), } V = \omega \sqrt{x^2 - 0} = \omega x$$

$$= v$$

(Maximum velocity)

at 'or' $y = \pm x$

$$\therefore V = \omega \sqrt{x^2 - x^2} = \omega \times 0 = 0$$

A particle vibrating in S.H.M passes with maximum velocity through the mean position and is rest at the extreme positions.

(iii) Acceleration \rightarrow Acceleration 'a' of the particle can be obtained by differentiating eqn (11) w.r.t time.

$$a = \frac{dV}{dt} = \frac{d}{dt} (v \cos \omega t) = v \frac{d}{dt} (\cos \omega t)$$

$$= v (-\sin \omega t) \frac{d}{dt} (\omega t) = -v \omega \sin \omega t$$

$$= -v \cdot \frac{v}{x} \sin \omega t$$

$$a = -\frac{v^2}{x} \sin \omega t \quad \text{--- (V)}$$

or In ΔOAP ,

$$\sin \omega t = \frac{OP}{OA} = \frac{y}{x} \quad \therefore a = -\frac{v^2}{x} \cdot \frac{y}{x}$$

$$\text{or } a = -\frac{v^2}{x^2} \cdot y \quad \text{--- (VI)}$$

since $v = x\omega$

$$a = -\frac{(x\omega)^2}{x^2} y = -\omega^2 y \quad \text{--- (VII)}$$

Eqn (V) gives the acceleration at different instants of time while eqn (VI) & (VII) gives the same for various displacements.

The -ve sign implies that the acceleration is directed towards the mean position. Since displacement (away from 0) is taken as a +ve quantity hence quantities directed towards 0 are taken as -ve. This is why acceleration is -ve.

$$\text{At } 0, y=0$$

$$a = -\omega^2 \times 0 = 0$$

$$y = \pm x$$

$$a = \pm \omega^2 x \quad (\text{Maximum acceleration})$$

A particle vibrating in S.H.M has zero acceleration while passing through mean position and has maximum acceleration while at extreme positions.

Since $\omega = \text{constant}$ (particle A rotating with uniform angular velocity)

From eqn (VII)

(i) acceleration $\propto y$ (displacement)

(ii) acceleration is always directed towards the mean position.

S.H.M can also be defined as the motion of a body is said to be s.h.m. if it moves in such a way that its acceleration is proportional to its displacement and is always directed towards mean position.

(iv) Time period (T) \rightarrow It is the time taken by the particle to complete one vibration.

If ' ω ' is the angular velocity of the particle, T is given by

$$T = \frac{2\pi}{\omega}$$

From eqn (VII) $\omega^2 = \frac{\text{acceleration}}{\text{displacement}}$

Rejecting -ve sign, we are taking only the magnitude

$$\omega = \sqrt{\frac{\text{acceleration}}{\text{displacement}}} \quad \text{or} \quad T = \frac{2\pi}{\sqrt{\frac{\text{acceleration}}{\text{displacement}}}}$$

$$\text{or} \quad T = 2\pi \sqrt{\frac{\text{displacement}}{\text{acceleration}}}$$

Knowing the displacement and acceleration of the particle, at a particular instant, its time period can be calculated.

Wave motion → Wave motion is a kind of disturbance that travels through a medium due to repeated vibrations of the particles of the medium about their mean position.

It is a means of transferring energy & momentum from one point to another of the medium, without any actual transportation of matter between the two points.

Wave motion is the transfer of energy and momentum from one point to the another point of the medium without actual transport of matter between two points. The medium of propagation. The dimensions in which a wave propagates energy. The energy transfer.

Waves are most commonly caused by wind. Wind-driven waves, or surface waves, are created by the friction between wind and surface water. As wind blows across the surface of the ocean or a lake, the continual disturbance creates a wave crest. The gravitational pull of the sun and moon on the earth also causes waves.

Wave motion is the disturbance that travels through the medium and is due to repeated periodic motion of the particles of the medium, the motion being handed over from particle to particle.

Characteristics of wave-motion

A wave motion has the following characteristics -
(i) The disturbance that travels through the medium is due to the repeated periodic motion of the particles of the medium.

(ii) Particles do not leave their mean positions but keep on vibrating to and fro, in S.H.M., about their mean positions.

(iii) The velocity of wave and the velocity of particles are different from each other. The velocity of the particles is variable. It is maximum when the particle passes through the mean position and is zero at extreme position. The velocity of wave is uniform.

(iv) Propagation of a wave results in formation of crests and troughs (in case of a transverse wave-motion) or in formation of compressions and rarefaction (in case of longitudinal wave).

(v) Energy is always carried in the direction of propagation of wave.

Two types of wave-motion

(i) Transverse wave motion \rightarrow In transverse wave-motion the particles of the medium are vibrating in a direction at right angles to the direction of propagation of wave. The medium gets raised above its normal level is called crest and depressed below the normal level is called trough.

(ii) Longitudinal wave-motion \rightarrow In longitudinal wave motion the particles of the medium vibrate in the direction of propagation of wave. The medium gets compressed ^{together} region is called compression region, while some other gets rarefied and is called a rarefaction region.

Example of transverse wave \rightarrow When we throw a pebble into the pond, we see the circular ripples formed on its surface which disappear gradually. The water moves up and down, and the effect, ripple, which is visible to us looks like an outwardly moving wave.

When you pluck the string of a guitar, the strings move up and down, exhibiting transverse wave. The sound wave is a longitudinal wave, but the wave on the guitar is however, a transverse wave. The particles in the string move perpendicular to the direction of the wave propagation.

Example of longitudinal wave \rightarrow A sound wave is a significant example of a longitudinal wave. When a speaker speaks some words in front of the microphone, he/she hit the air thousands of times per second at different frequencies. The sound particles travel along with the air particles and enter the mic to produce sound.

When we clap while singing a song, our hand produces that familiar sound of a clap. When we applaud, we compress and displace the air particles between our hands for a part of a second, while produces the sound of a clap we are familiar with.

Comparison between Transverse & Longitudinal Wave

Transverse wave

- 1) The particles move at right angles to the direction of wave propagation.
- 2) They are possible only in solids.
- 3) They consist of crests & troughs.
- 4) They can be polarised.
- 5) Vibrations in a string is an example of transverse wave. (light wave)
- 6) There are no pressure variations.

Longitudinal wave

- 1) The particles of medium vibrate in the same direction.
- 2) They are possible in all kinds of media.
- 3) They consist of regions of compression & rarefaction.
- 4) They cannot be polarised.
- 5) Sound waves in air is an example of longitudinal wave.
- 6) There is a pressure variation throughout the medium.

Definition of different wave parameters

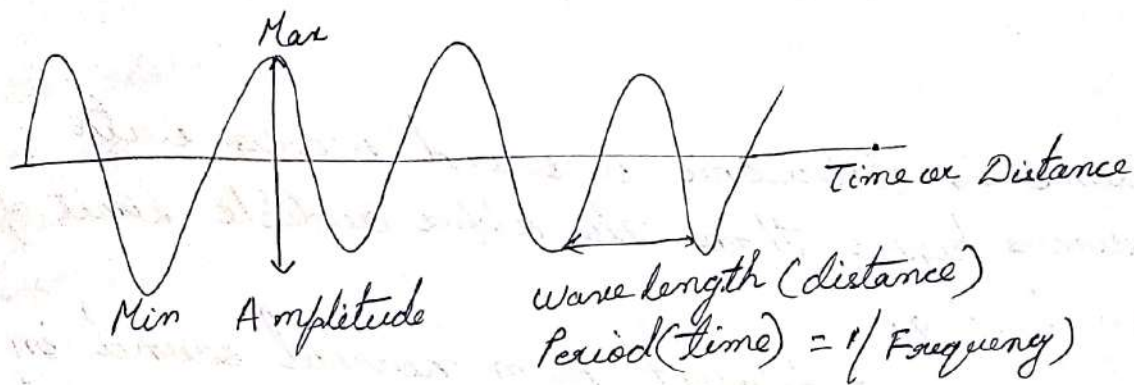
Amplitude (A) → The height of the wave. Equal to the $(\text{maximum} - \text{minimum})/2$. For sound this is measured in decibels (dB) or pascals (Pa). Normal values are 1-3 megapascals for clinical ultrasound. The term transmit voltage is also sometimes used to describe amplitude and is measured in volts. Amplitude shows up as a change in the brightness on the doppler tracings.

Wavelength (λ) → The length/distance of one cycle. Clinical ultrasound uses 0.15 to 0.8 mm wavelength. Shorter wavelengths produce higher resolution pictures.

It is determined by the source of ultrasound (with a given f) and the medium (with a given velocity of propagation).

Acoustic variables are those things which change due to a mechanical interaction of the sound wave with a medium and include pressure, density, temperature and particle motion.

Wave parameters are the group of characteristics that identify a wave. They are shown in the figure. From these components, one can usually derive other wave properties (i.e., period, power & intensity) based on known equations.



Frequency (f) \rightarrow The number of cycle per second, measured in Hertz (Hz). Clinical ultrasound uses a frequency of 2-10 Megahertz. The range of human hearing spans 20Hz to 20 kilohertz.

Time Period (t) \rightarrow The reciprocal of frequency ($1/f$) which is the time for one cycle. Clinical ultrasound uses periods of 0.1 to 0.5 microseconds. Both frequency & period are determined by the source of ultrasound only.

Propagation velocity (c) \rightarrow How fast the wave is moving in the specific medium.

Propagation velocity is dependent on the medium and is related to the stiffness of the medium and something called the bulk modulus (directly related to how stiff the medium is) by the equation. $c = \text{square root} (\text{bulk modulus} / \text{density})$

Derivation of Relation between Velocity, Frequency and wavelength of a wave \rightarrow

$$\text{Wave velocity} = \frac{\text{Distance}}{\text{Time}}$$

$$\Rightarrow v = \frac{\lambda}{T}$$

$$\Rightarrow v = v\lambda \left[\because \frac{1}{T} = \text{Frequency} \right]$$

$$\therefore \text{Wave velocity} = \text{Frequency} \times \text{Wavelength}$$

Ultrasonic \rightarrow Ultrasonic is sound waves with frequencies higher than the upper audible limit of human hearing.

Ultrasound is not different from normal sound in its physical properties, except that human cannot hear it. This limit varies from person to person and is approximately 20 kilohertz. The term ultrasonic is applied to ultrasound waves of very high amplitudes.

Properties \rightarrow 1) The ultrasonic waves cannot travel through vacuum.

2) These waves travel with speed of sound in a given medium.

3) Their velocity remains constant in homogeneous media.

4) These waves can weld certain plastics, metals etc.

5) These can produce vibrations in low viscosity liquids.

6) The ultrasonic waves are reflected and refracted just like light waves i.e.,

(a) Angle of incidence is equal to angle of reflection.

(b) Incident ray, reflected ray and normal lie in same plane.

(c) If 'i' is angle of incidence, 'r' is angle of refraction, v_1 is velocity of ultrasonic wave in incident medium and v_2 in refracted medium then

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2}$$

This is called Snell's law of refraction.

7) The speed of ultrasonic waves / acoustic waves is more in more dense media i.e; $v_g < v_1 < v_2$. If ultrasonic waves enter from rarer medium to denser medium, then $v_1 < v_2$ so equation gives $\sin i / \sin r < 1 \Rightarrow i < r$. Thus ultrasonic waves will bend away from normal. Similarly when ultrasonic waves enter from denser to rarer medium, then they bend toward normal. This property is just opposite to that of light.

8) As ultrasonic waves cannot travel through vacuum, therefore if these waves travel through a non-homogeneous medium, then at each discontinuity like crack or change in density or presence of impurity etc. the amplitude and thus intensity of ultrasonic waves decreases by some amount. This decrease in intensity of ultrasonic waves as they travel through a medium is called Attenuation. The vacuum in the material causes strong reflection of ultrasonic waves while impurities or discontinuity cause the scattering of ultrasonic waves leading to net decrease in intensity. The attenuation is increased with increase in frequency of ultrasonic waves for a given medium. The intensity of ultrasonic waves decreases exponentially according to the relation.

$$I = I_0 e^{-ax}$$

where I_0 = Intensity at surface

I = Intensity at depth x inside the sample

a is called Monochromatic Attenuation coefficient. Its value is different for different media and for a given medium, its value is different for different frequencies/wavelengths.

With increase in frequency of ultrasonic waves, value of a also increases.

UNIT - 7 Heat & Thermodynamics

Heat is a form of energy it is measured in calorie and kilocalorie. It flows from one body to another body on account of temperature difference.

1 Calorie is the amount of heat required to raise the temperature of 1 gram of water through 1°C .

1 kilo calorie = 1000 calorie.

Temperature \rightarrow The degree of hotness or coldness of a body is called temperature.

Difference between Heat & Temperature

Heat (Q)

Heat is the amount of energy in a body.

Total kinetic and potential energy contained by molecules in an object.

Flows from hotter object to cooler object.

The unit is Joule and is measured by calorimeter.

Heat is a form of internal energy obtained because of the random motion and the attractive force of molecules within the substance.

Temperature (T)

Temperature is the measure of the intensity of heat.

Average kinetic energy of molecules in a substance.

Rises when heated and falls when cooled.

The unit is kelvin & is measured by thermometer.

Temperature is a quantity that determines which direction the heat energy will flow in.

The unit of heat in S.I is Joule.
The amount of energy needed to raise the temperature of a given mass by one degree.

C.G.S \rightarrow (1 Calorie) The heat energy needed to increase the temperature of 1 gm of water by one degree Celsius.
F.P.S \rightarrow (B.T.U) \rightarrow

M.K.S \rightarrow (Joule) instead of Calorie.

Specific heat \rightarrow It is observed that more amount of heat is required to rise the temperature of a greater mass substance i.e; $Q \propto m$. - (i)

It is also observed that a greater amount of heat is required to rise the temperature of a body through higher range i.e; $Q \propto \Delta\theta$. - (ii)
where $\Delta\theta$ is the change in temperature.

Combiningly, $Q \propto m \Delta\theta$
 $\Rightarrow Q = S \cdot m \Delta\theta$

where 'S' is the specific heat of the substance

If $m=1g$, $\Delta\theta=1^\circ$ then $Q=S$

Therefore, specific heat of a substance is defined as the amount of heat required to raise the temperature of unit mass substance through 1° .

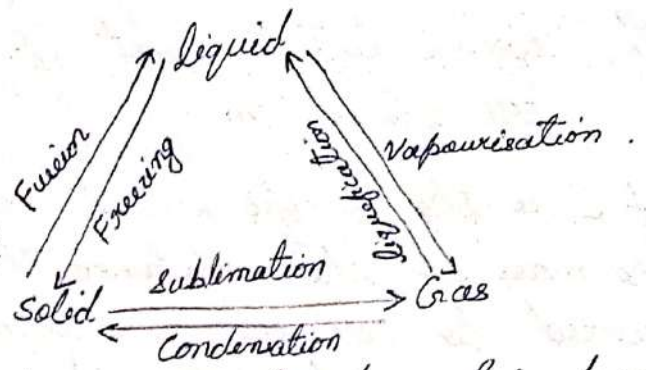
The specific heat of water is 1 Calorie / gm / $^\circ$ C.

Unit \rightarrow Calorie / gm / $^\circ$ C.

Dimensional Formula $\rightarrow S = \frac{Q}{m \Delta\theta} = \frac{[ML^2T^{-2}]}{[M][K]} = [L^2T^{-2}K^{-1}]$

Change of state

Sublimation → The process in which a solid changes into vapour without changing into liquid and from vapour changes into solid without changing into liquid is known as sublimation.



For example - Naphthalene balls, NH_4Cl , Iodine, Dry ice, Camphor.

Latent heat → Latent heat of a substance is defined as the amount of heat required to convert a substance from its one state to another state without any change of temperature.

Specific latent heat → Specific latent heat of a substance is defined as the amount of heat required to convert 1gm of substance from its one state to another state without any change of temperature.

There are two types of specific latent heat →

(i) Specific latent heat of fusion → Specific latent heat of fusion of a solid is defined as the amount of heat required to convert 1gm of solid into its liquid at its melting point without any change of temperature.

(ii) Specific latent heat of vapourisation → Specific latent heat of vapourisation of a liquid is defined as the amount of heat required to convert 1gm of liquid into its vapour at its boiling point without any change in temperature.

The specific latent heat of vapourisation of water is 540 caloric/gm.

If Q is the specific latent heat of a substance m is the mass of the substance then, the amount of heat required to convert a substance from one state to another state is given by $Q = ml$ $l = \frac{Q}{m}$

Unit \rightarrow Joules/kg

Dimensional Formula $\rightarrow Q = ML^2T^{-2}$ $m = M$

$$l = \frac{Q}{m} = \frac{ML^2T^{-2}}{M} = L^2T^{-2}$$

Thermal Expansion \rightarrow The heat capacity of a body is defined as the amount of heat required to raise the temperature of the body through 1°C .

It is given by the product of the mass of the body and its specific heat i.e; Heat Capacity = ms .

Unit \rightarrow Caloric/ $^\circ\text{C}$.

Thermal expansion is the tendency of matter to change its shape, area and volume in response to a change in temperature. Temperature is a monotonic function of the average molecular kinetic energy of a substance. When a substance is heated, the kinetic energy of its molecule increases. Thus, the molecules begin vibrating more and usually maintain a greater average separation. Materials which contract with increasing temperature and unusual; this effect is limited in size, and only occurs within limited temperature ranges. The relative expansion

(also called strain) divided by the change in temperature is called the material's coefficient of thermal expansion and generally varies with temperature.

$$\frac{\Delta L}{L} = \alpha_L \Delta T$$

ΔL = change in length

L = original length

ΔT = change in temperature

α_L = linear coefficient of thermal expansion.

Expansion of solids \rightarrow The expansion of solid refers to the example of gap in the railway tracks which during the summer expands and contracts in winter.

Coefficient of linear, superficial and cubical expansion of solids (Relation between α , β & γ .)

Heat and work are two different ways of transferring energy from one system to another. The distinction between heat and work is.

Heat is the transfer of thermal energy between systems while work is the transfer of mechanical energy between two systems.

Heat is the energy associated with the random motion of particles, while work is the energy of ordered motion in one direction. Therefore heat is low-quality energy & work is high quality energy.

Work (Mechanical)	Heat (Thermal)
Force \times displacement	Temperature difference

Relationship between internal energy, heat and work
The change in internal energy is given by the equation

$$q = \Delta E + w$$

$$\Delta E = q - w$$

or

$$\Rightarrow q - P\Delta V$$

There are certain conditions under which heat and work terms become definite:

constant pressure and
constant volume.

Energy, work & heat

$$\Delta E = w + q$$

$$w = -P\Delta V$$

(E) the ability to do work (w) or produce heat (q)

+w = work is done ON the system
(compression)

-w = work is done BY the system
(expansion)

+q = heat is absorbed

-q = heat is released.

Whenever heat is converted into work or work into heat, the quantity of energy disappearing in one form is equivalent to the quantity of energy appearing in the other.

If an amount of work 'W' results in the production of an amount 'H' of heat

$$W \propto H \text{ or } W = JH$$

where J is a constant called Joule's mechanical equivalent of heat.

$$\text{If } H=1, W=J$$

Joule's mechanical equivalent of heat is defined as the amount of work required to produce a unit quantity of heat.

$$J = \frac{W}{H}$$

Value of J is

$$J = 4.2 \times 10^7 \text{ erg cal}^{-1} = 4.2 \text{ J cal}^{-1}$$

J should be treated only as a conversion factor from heat to work and vice-versa.

First law of Thermodynamics \rightarrow It states that if the quantity of heat supplied to a system is capable of doing work, then the quantity of heat absorbed by the system is equal to the sum of the increase in the internal energy of the system, and the external work done by it.

If ' Q ' is the amount of heat added to the system and U_1 is the initial energy of the system,

$$\text{Then, The total energy} = U_1 + Q.$$

After the system gets heated the final energy is U_2 and some work ' W ' is done by the gas.

$$\text{then, The total energy} = U_2 + W.$$

According to the law of conservation of energy

$$U_1 + Q = U_2 + W.$$

$$Q = (U_2 - U_1) + W$$

$$\Rightarrow \boxed{dQ = dU + dW}$$

UNIT-8 Optics

The branch of physics which deals with light is called optics. Light is a transverse wave. It obeys different properties like reflection, refraction, dispersion, diffraction, polarisation etc.

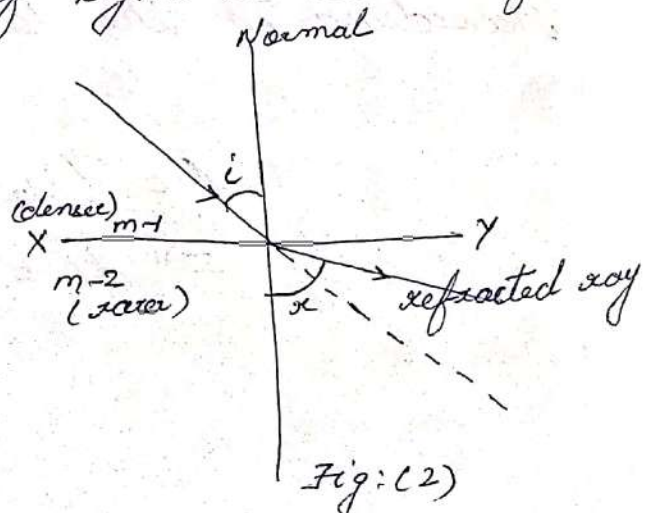
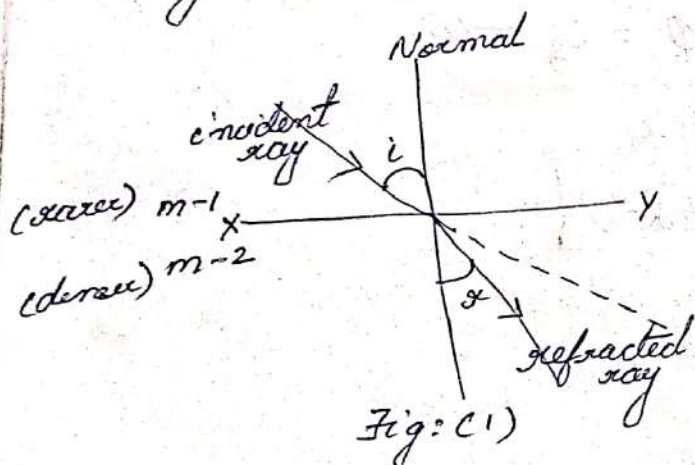
Reflection \rightarrow It is the property of light by virtue of which light is sent back to the same medium from which it is coming after striking on a shining surface.

Most of the substances are reflecting but mirror is the only substance which can reflect approximately 100% of light incident upon it.

Laws of reflection \rightarrow There are two laws of reflection.

- (i) The incident ray, the reflected ray and the normal to the reflecting surface at the point of incidence all lie in the same plane.
- (ii) The angle of incidence is equal to the angle of reflection. i.e., $\angle i = \angle r$.

Refraction \rightarrow When light travels from one medium to another medium it bends as well as its velocity changes. This phenomenon of light is called refraction.



When light travels from rarer to denser medium it bends towards the normal and its velocity decreases.

When light travels from denser to rarer medium it bends away from the normal and its velocity increases.

Laws of refraction \rightarrow There are two laws of refraction.

(i) The incident ray, the refracted ray and the normal to the refracting surface all lie in the same plane.

(ii) The ratio between the sine of the angle of incidence to the sine of the angle of refraction is a constant, i.e.;

$$\frac{\sin i}{\sin r} = \text{constant} \\ = \frac{\mu_2}{\mu_1}$$

This is also called as Snell's law; where μ_2 is the refractive index of the second medium and μ_1 is the refractive index of the first medium.

Refractive index \rightarrow Refractive index of a medium is defined as the ratio between the velocity of light in vacuum to the velocity of light in that medium.

$$\text{i.e.}, \boxed{\mu = \frac{c}{v}}$$

Since, it is a pure ratio between two velocities so it has no unit and no dimension.

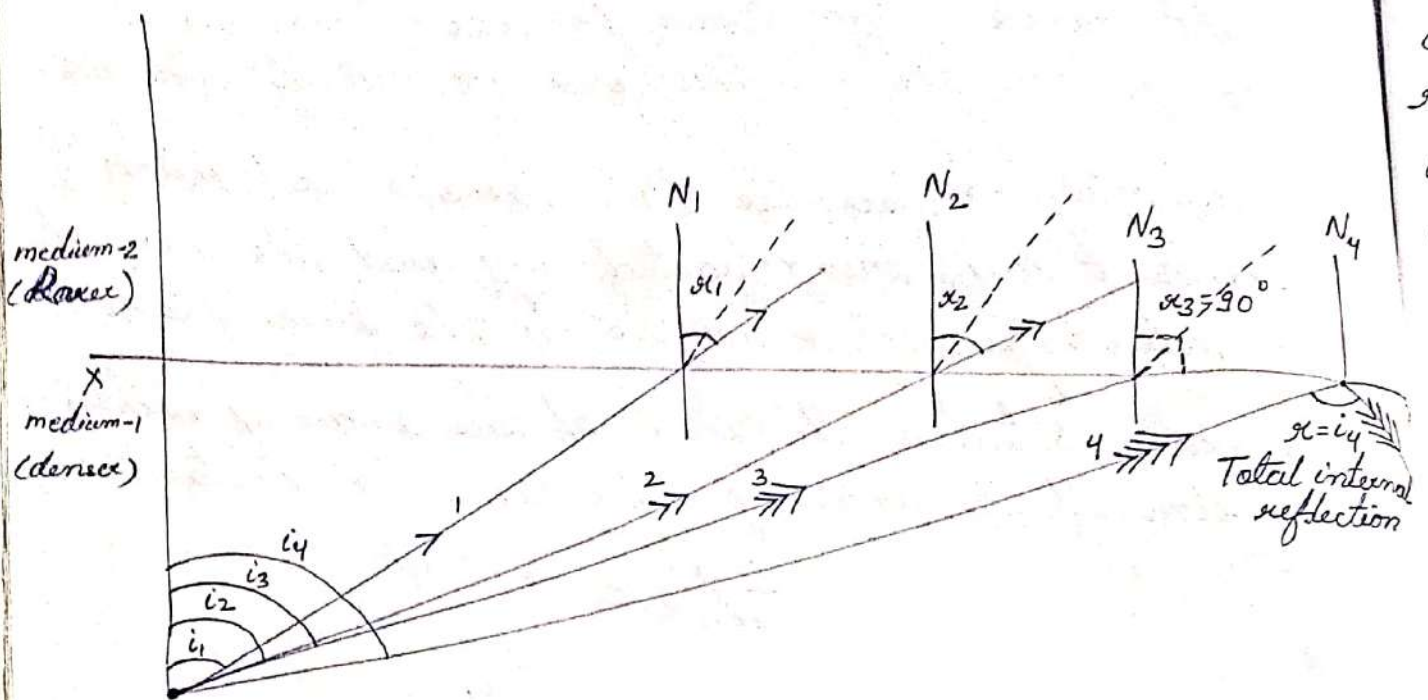
$$\text{For air, } \mu = \frac{c}{c} = 1$$

$$\text{For any other medium } \boxed{v < c} \quad \boxed{\mu > 1}$$

The refractive index of glass is 1.5 and water is 1.33

The refractive index of red light is less than the refractive index of violet light i.e. $\boxed{\mu_R < \mu_V}$ because the velocity of red light is greater than the velocity of violet light.

Total internal reflection



Consider a refracting surface XY which separates two mediums: med-1 (denser) and med-2 (rarer), of source of light 'S' is placed in the denser medium. The ray of light which touches the surface perpendicularly goes undeviated. For ray-1, the angle of incidence ' i_1 ' and angle of refraction is ' r_1 '. Since the ray of light is travelling from denser to rarer so it bends away from the normal and the angle of refraction is always greater than the angle of incidence. If we increase the angle of incidence, then the angle of refraction also increases as shown for the ray-2. For a particular value of angle of incidence the angle of refraction in the rarer medium is 90° . This value of angle of incidence is called critical angle. $\alpha_c < \alpha_i$

Critical angle is defined as the angle of incidence in the denser medium for which the angle of refraction in the rarer medium is 90° .

If we further increase the angle of incidence beyond critical angle then the ray instead of being refracted, reflects back to the same medium obeying the laws of reflection. This phenomenon of light is called total internal reflection.

Total internal reflection is the phenomenon of light by virtue of which the light travelling from denser to rarer instead of being refracted returns back to the same medium obeying the laws of reflection provided the angle of incidence in the denser medium is greater than the critical angle.

Applying Snell's law, $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$

Here, for $i = c$, $r = 90^\circ$

μ_1 is the refractive index of the first medium.
 $= \mu$ (say).

μ_2 is the refractive index of the rarer medium.
Let the rarer medium is air medium
Then $\mu_2 = 1$

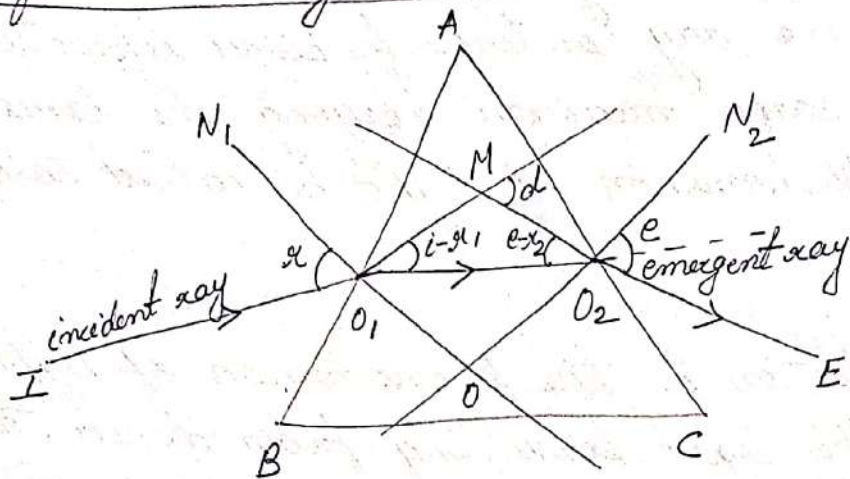
Substituting the values.

$$\frac{\sin c}{\sin 90^\circ} = \frac{1}{\mu}$$

$$\Rightarrow \mu = \frac{1}{\sin c}$$

This is the relation between refractive index and critical angle.

Refraction through Prism



Consider a glass prism ABC . IO_1 is the incident ray to the surface AB . O_1O_2 is the refractive ray inside the prism. This ray again refracts at the surface AC & O_2E is the final emergent ray. N_1O & N_2O are the normals to the surfaces AB & AC respectively. ' i ' is the angle of incident and ' e ' is the angle of emergent. α_1, α_2 are the angle of refraction.

Let us extend the incident ray in the forward direction and emergent ray in the backward direction they meet at a point M . The angle between them is known as angle of deviation and is denoted by d .

In the $\triangle MO_1O_2$ ' d ' is the external angle.

$$\therefore d = (i - \alpha_1) + (e - \alpha_2) \\ = (i + e) - (\alpha_1 + \alpha_2) \quad \text{--- (i)}$$

In the quadrilateral AO_1O_2 , $\angle A + \angle O = 180^\circ$ --- (ii)

In the $\triangle O_1O_2$, $\alpha_1 + \alpha_2 + \angle O = 180^\circ$ --- (iii)

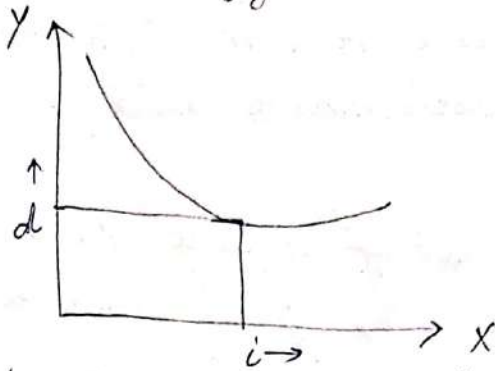
Comparing eqn (ii) & (iii) $A = \alpha_1 + \alpha_2$ --- (iv)

Using in eqn (i) $d = i + e - A$

$$\Rightarrow \boxed{A + d = i + e} \quad \text{--- (v)}$$

For different values of angle of incidence, we have different values of angle of deviation.

A graph is plotted between i vs d . Taking angle of incidence along x -axis and angle of deviation along y -axis as shown in the fig:



From the graph it is clear that initially with an increase in the angle of incidence the angle of deviation decreases. For a particular value of angle of incidence the angle of deviation is minimum, called as angle of minimum deviation (d_m). Further increase in the angle of incidence, the angle of deviation also increases.

At the point of minimum deviation the following conditions are satisfied:

- (i) $i = e$
- (ii) $r_1 = r_2 = (\alpha) (2\alpha)$

Substituting in eqn (V)

$$A + d_m = i + i$$

$$\Rightarrow 2i = A + d_m$$

$$\Rightarrow \boxed{i = \frac{A + d_m}{2}} \quad \text{--- (VI)}$$

Using in eqn (IV)

$$A = r + r$$

$$\Rightarrow \alpha = \frac{A}{2} \quad \text{--- (VII)}$$

This is the formula used to determine the refractive index of the material of prism.

From Snell's law $\frac{\sin i}{\sin r} = \mu$

$$\Rightarrow \mu = \frac{\sin \frac{A + d_m}{2}}{\sin \frac{A}{2}}$$

Fiber Optics \rightarrow Fiber optics, or optical fiber, refers to the medium and the technology associated with the transmission of information as light pulses along a glass or plastic strand or fiber. Fiber optics is used long distance and high performance data networking.

Optical fibers consist of a pure glass core surrounded by multiple layers. The first layer is a reflective cladding, which acts like a long, flexible mirror, reflecting light along the length of the glass core. This principle is known as total internal reflection.

Application of fiber optics \rightarrow

- (i) These are used to study the interior of lungs and other parts of body which cannot be viewed directly otherwise.
- (ii) These are used for the study of tissues and blood vessels far below the skin.
- (iii) It can be used to transmit high intensity laser light inside the body for medical purposes.
- (iv) They are used in the field of communication in sending video signals from one place to other.

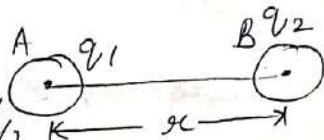
UNIT-9 Electrostatic & Magnetostatics

Electrostatic \rightarrow If the charge is at rest then the branch of physics dealing with it is called electrostatic.

Coulomb's law \rightarrow Like charges repel each other and unlike charges attract each other. The magnitude of the force of attraction or repulsion between two charges is given by Coulomb's law.

statement \rightarrow It states that the magnitude of the force of attraction or repulsion between two charges (charged bodies) is directly proportional to the product of their charges and inversely proportional to the square of the separation between them.

Explanation \rightarrow Consider two bodies A & B having charges q_1 & q_2 respectively, separated by a distance ' x ' from each other. If ' F ' be the force of attraction or repulsion between them. Then, according to the statement



(i) $F \propto q_1 q_2$

(ii) $F \propto \frac{1}{x^2}$

Combining, $F \propto \frac{q_1 q_2}{x^2}$

$\Rightarrow F = k' \frac{q_1 q_2}{x^2}$

where ' k ' is a constant.

The above equation represents the Coulomb's law in scalar form.

One coulomb is the amount of charge which when separated from another similar charge by a distance of 1m in air, experiences a repulsive force of 9×10^9 N.

Value of k'

(a) In C.G.S $\rightarrow k' = \frac{1}{k}$ where, k is the dielectric constant of the medium.

For air/vacuum $k=1$, so, the coulomb's law in C.G.S is given by $F = \frac{1}{k} \frac{q_1 q_2}{x^2}$ — (i) (For any medium)

$$F = \frac{q_1 q_2}{x^2} \quad (\text{For air/free space}) \quad \text{--- (ii)}$$

(b) In S.I $\rightarrow k' = \frac{1}{4\pi\epsilon}$

where ϵ is the permittivity of the medium.

For air/free space $k' = \frac{1}{4\pi\epsilon_0}$

Therefore, the coulomb's law in S.I is given by

$$F = \frac{1}{4\pi\epsilon} \frac{q_1 q_2}{x^2} \quad \text{--- (iii)} \quad (\text{For any medium})$$

$$\& \quad F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{x^2} \quad \text{--- (iv)} \quad (\text{For air/free space})$$

The value of $\epsilon_0 = 8.854 \times 10^{-12}$

$$k' = \frac{1}{4\pi\epsilon_0} = \frac{1}{4 \times 3.14 \times 8.854 \times 10^{-12}} \\ \approx 9 \times 10^9$$

So, the coulomb's law in S.I for free space can be written as $F = 9 \times 10^9 \frac{q_1 q_2}{x^2}$

The ratio between the permittivity of a medium to the permittivity of the free space is called relative permittivity and its denoted by ϵ_r .

$$\text{i.e.; } \epsilon_r = \frac{\epsilon}{\epsilon_0} \Rightarrow \boxed{\epsilon = \epsilon_r \epsilon_0} \quad \text{--- (v)}$$

So, the Coulomb's law in S.I may be written as

$$F = \frac{1}{4\pi\epsilon_0\epsilon_r} \frac{q_1q_2}{x^2} \quad \text{--- (VI)} \quad (\text{For any medium})$$

Unit of ϵ

We know, $F = \frac{1}{4\pi\epsilon} \frac{q_1q_2}{x^2}$

$$\Rightarrow \epsilon = \frac{q_1q_2}{4\pi Fx^2} \quad \text{--- (VII)}$$

So, the unit of ϵ is $C^2N^{-1}m^2$ and the dimensional formula of ϵ is $\frac{[AT][AT]}{[MLT^{-2}][L^2]} = [M^{-1}L^{-3}T^4A^2]$.

Dielectric Constant (κ) \rightarrow Dielectric constant of a medium is defined as the ratio between the force between two charges separated by some distance in air to the force between the same two charges separated by the same distance in that medium. i.e; $\kappa = \frac{F_{air}}{F_{medium}}$

$$= \frac{\frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{x^2}}{\frac{1}{4\pi\epsilon} \frac{q_1q_2}{x^2}} = \frac{\epsilon}{\epsilon_0} = \epsilon_r$$

$$\Rightarrow \boxed{\kappa = \epsilon_r}$$

Therefore, dielectric constant of a medium is numerically equal to relative permittivity of the medium.

It has no dimension.

The value of air medium for $\boxed{\kappa = 1}$
 $\boxed{\kappa > 1}$ (For other medium).

Unit of charge

(a) In C.G.S \rightarrow

(i) Electrostatic unit \rightarrow stat coulomb
(e.s.u)

$$F = \frac{q_1 q_2}{r^2} \text{ (For air medium)}$$

If $F = 1 \text{ dyne}$, $q_1 = q_2 = q$, $r = 1 \text{ cm}$
Then, $q^2 = (1 \text{ statc})^2$

$$\boxed{q = 1 \text{ statc}}$$

Therefore, 1 stat coulomb charge is the amount of charge which when separated from another similar charge by a distance of 1 cm in air medium, experiences a repulsive force of one dyne.

(ii) Electromagnetic unit \rightarrow The electromagnetic unit of charge is e.m.u of charge / ab coulomb.

(b) In S.I \rightarrow The S.I unit of charge is coulomb.

$$\text{In S.I } F = \frac{9 \times 10^9 q_1 q_2}{r^2} \text{ (For air/vacuum)}$$

If $F = 9 \times 10^9 \text{ N}$, $q_1 = q_2 = q$, $r = 1 \text{ m}$

$$\text{Then, } q^2 = (1 \text{ C})^2$$

$$\Rightarrow q = \pm 1 \text{ C}$$

Therefore, 1 coulomb charge is the amount of charge which when separated from another similar charge by a distance of 1 m in air, experiences a repulsive force of $9 \times 10^9 \text{ N}$.

Relation between coulomb & stat coulomb

Let $1C = x$ stat coulomb

Consider two charges each of $1C$, separated by a distance $1m$ the force between them in S.I is given by

$$F = 9 \times 10^9 \frac{1 \times 1}{1^2} \\ = 9 \times 10^9 N - (1)$$

In C.G.S

$$F = \frac{9 \times 9 \times x^2}{(10^2)^2} \\ = \frac{x \times x}{(10^2)^2} \text{ dyne}$$

$$= \frac{x^2}{10^4} \times \frac{1}{10^5} N = \frac{x^2}{10^9} N - (2)$$

$$\Rightarrow \frac{x^2}{10^9} = 9 \times 10^9$$

$$\Rightarrow x^2 = 9 \times 10^{18}$$

$$\Rightarrow x = \sqrt{9 \times 10^{18}}$$

$$\Rightarrow x = 3 \times 10^9$$

$$\Rightarrow 1C = 3 \times 10^9 \text{ stat coulomb}$$

Relation between coulomb & ab coulomb

$$1 \text{ coulomb} = \frac{1}{10} \text{ ab coulomb}$$

Dimensional formula of charge = $[AT]$.

Electric Potential →

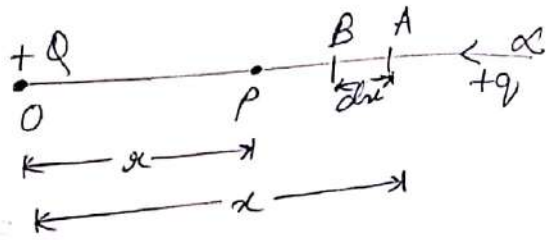
(Electric field Potential) → Electric field potential at a point within an electric field is defined as the amount of work done in bringing a unit +ve charge from infinity to that point.

$$\text{i.e.; } \boxed{V = \frac{W}{+q}}$$

It is a scalar quantity.

Expression for Electric field Potential

Consider a charge '+Q' placed at 'O'. 'P' be a point within its field at '+Q', distance of 'x' from 'O'. Let us consider a '+q' charge at ∞.



The amount of work done in bringing '+q' charge from ∞ to the point 'P' is W.

Let A & B are the two instantaneous positions very close to each other separated by a small distance 'dr', at a distance of 'r' from the point O.

Let dW be the small amount of work done due to the small amount of displacement dr from A to B against electric field.

$$\begin{aligned} \text{Then, } dW &= \vec{F} \cdot \vec{dr} \\ &= F \cdot dr \cdot \cos 180^\circ \\ &= -F \cdot dr \\ &= -\frac{kQq}{r^2} \cdot dr \quad \text{--- (1)} \end{aligned}$$

Now the total amount of work done in bringing +q charge from ∞ to the point P can be obtained by integration. i.e., $W = \int_0^W dw$

$$= \int_{\infty}^r \frac{-k'Qq}{x^2} dx$$

$$= -k'Qq \left[-\frac{1}{x} \right]_{\infty}^r$$

$$= -k'Qq \left[\frac{1}{r} - \frac{1}{\infty} \right]$$

$$= \frac{k'Qq}{r} \quad \text{--- (ii)}$$

Now, the potential at the point P $V = \frac{W}{+q}$

$$= \frac{k'Qq/r}{+q}$$

$$\Rightarrow \boxed{V = \frac{k'Q}{r}} \quad \text{--- (iii)}$$

This is the expression for electric field potential at a point due to a charge +Q at a distance of 'r' from it.

Units

(i) C.G.S system

(a) E.S.U \rightarrow stat volt

$$1 \text{ stat volt} = \frac{1 \text{ erg}}{1 \text{ stat coulomb}}$$

Potential at a point is said to be 1 stat volt if 1 erg of work is done in bringing 1 stat coulomb charge from ∞ to that point against the electric field.

(b) E.M.U \rightarrow ab volt

$$1 \text{ ab volt} = \frac{1 \text{ erg}}{1 \text{ ab coulomb}}$$

The potential at a point is said to be 1 ab volt if 1 erg of work is done in bringing 1 ab coulomb charge from ∞ to that point against the electric field.

(ii) S.I unit \rightarrow volt

$$1 \text{ volt} = \frac{1 \text{ Joule}}{1 \text{ coulomb}}$$

The potential at a point is said to be 1 volt if 1 Joule of work is done in bringing 1 coulomb charge from infinity to that point against the electric field.

Relation between volt and stat volt

$$1 \text{ volt} = \frac{1 \text{ Joule}}{1 \text{ coulomb}}$$

$$1 \text{ V} = \frac{10^7 \text{ erg}}{3 \times 10^9 \text{ stat C.}}$$

$$1 \text{ V} = \frac{1}{300} \text{ stat volt}$$

Relation between volt & ab volt

$$1 \text{ volt} = \frac{1 \text{ Joule}}{1 \text{ coulomb}}$$

$$= \frac{10^7 \text{ erg}}{10}$$

$$\frac{1}{10} \text{ ab coulomb}$$

$$1 \text{ V} = 10^8 \text{ ab volt}$$

Dimensional formula

$$V = \frac{W}{+q}$$

$$= \frac{[ML^2T^{-2}]}{[AT]} = [ML^2T^{-3}A^{-1}]$$

Potential difference (V)

The electric field potential at different points within an electric field are different.

Let us consider r_A & r_B from the point source charge $+Q$. Then the potential at $V_A = \frac{kQ}{r_A}$ and at B, $V_B = \frac{kQ}{r_B}$.

$$\text{Let } r_A > r_B$$

$$\text{Then, } V_A < V_B.$$

$$\text{The difference } V_B - V_A = \frac{kQ}{r_B} - \frac{kQ}{r_A}$$

$$= kQ \left[\frac{1}{r_B} - \frac{1}{r_A} \right] \text{ is called potential difference.}$$

If these two points A & B are connected to each other by a conductor then +ve charge goes from higher potential to lower potential and the negative charge flows from higher lower potential to higher potential.

Electric field \rightarrow Electric field of an electric charge is the space or region surrounding the charge within which it can influence other charges.

It is a subjective idea or it is qualitative concept.

Electric field intensity (\vec{E}) \rightarrow Electric field intensity at a point within an electric field is defined as the amount of force experienced by a unit positive charge placed at that point.

$$\text{i.e.; } \boxed{\vec{E} = \frac{\vec{F}}{+q}}$$

It is a vector quantity. The magnitude of electric field intensity $E = \frac{kQq}{x^2} / +q$

$$\boxed{E = \frac{kQ}{x^2}}$$

Units \rightarrow (a) In C.G.S \rightarrow Dyne/stat C.

(b) In S.I \rightarrow Newton / Coulomb

$$\text{Dimensional formula } \rightarrow \frac{[MLT^{-2}]}{[AT]} = [MLT^{-3}A^{-1}]$$

Capacitance \rightarrow Capacitance is the capacity of a capacitor to store charge.

Capacity \rightarrow Whenever some additional charge is given to a conductor then its potential increases i.e.; $Q \propto V$
 $\Rightarrow Q \propto CV - \textcircled{1}$

where C is called capacity of the conductor.

$$\text{From eqn } \textcircled{1} \quad C = \frac{Q}{V}$$

Therefore, capacity of a conductor may be defined as the ratio between the total charge on the conductor to its potential.

Therefore, capacity of a conductor may be defined as the amount of additional charge required to raise its potential by 1 unit.

Units

O.S.I → Farad

$$1 \text{ Farad} = \frac{1 \text{ coulomb}}{1 \text{ volt}}$$

Therefore, capacity of a conductor is said to be 1 Farad if 1 coulomb of additional charge increases its potential by 1 volt.

(2) In C.G.S

(a) Electrostatic unit → stat Farad

$$1 \text{ stat Farad} = \frac{1 \text{ stat coulomb}}{1 \text{ stat volt}}$$

Therefore, capacity of a conductor is said to be 1 stat farad if 1 stat coulomb ~~is sufficient to increase its potential by 1 stat volt~~ ~~to someone who~~

~~is sufficient to increase its potential by 1 stat volt.~~

(b) Electromagnetic unit → ab Farad

$$1 \text{ ab farad} = \frac{1 \text{ ab coulomb}}{1 \text{ ab volt}}$$

Therefore, capacity of a conductor is said to be 1 ab farad if a charge of 1 ab coulomb is sufficient to increase its potential by 1 ab volt.

Relation between farad & stat farad

$$\begin{aligned} 1 \text{ farad} &= \frac{1 \text{ coulomb}}{1 \text{ volt}} \\ &= \frac{3 \times 10^9 \text{ stat C}}{\frac{1}{300} \text{ stat V}} \\ &= 9 \times 10^{11} \text{ stat farad} \end{aligned}$$

Relation between farad & ab farad

$$\begin{aligned} 1 \text{ ab farad} &= \frac{1 \text{ C}}{1 \text{ V}} \\ &= \frac{10^9 \text{ abc}}{10^9 \text{ ab V}} \\ &= \frac{1}{10^9} \text{ ab V.} \end{aligned}$$

Dimensional formula of capacity

$$\begin{aligned} \text{Since, } C &= \frac{Q}{V} \\ &= \frac{[AT][AT]}{[ML^2T^{-2}]} = [M^{-1}L^{-2}T^4A^2] \end{aligned}$$

Capacitor \rightarrow Capacitor is an electrical device which can store charge and increases the capacity of a conductor upto infinite times. Its circuit symbol is $\text{--}\text{||}\text{--}$

Principle of capacitor

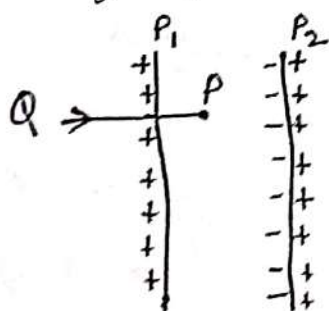


fig: (a)

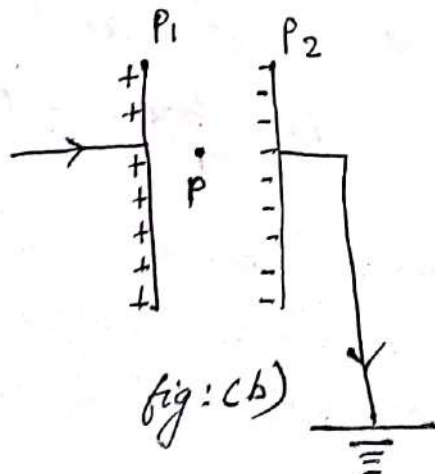


fig: (b)

Consider two parallel plates P_1 & P_2 . Let P_1 is given +ve charge 'Q' and let P_2 is neutral.

By the method of induction by -ve charges of the plate P_2 are towards P_1 (and the -ve charges of the plate P_2 are towards P_1) and the +ve charges of the plate P_2 are on the outer surface.

P be a point in between the two plates. Let the potential at the point P is 'V'. The case is shown in the fig: (a)

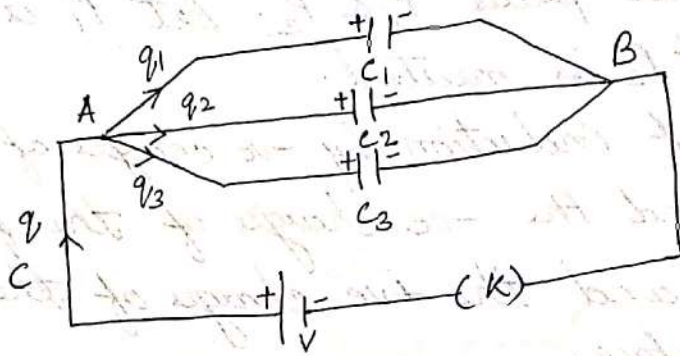
Now, the plate P_2 is earthed as shown in fig: (b). The +ve charges flows to the earth and the plate P_2 becomes -ve. Now in the presence of this plate, the work done in bringing a unit +ve charge from infinity to the point P will be less. Thus the potential at the point P decreases keeping the charge 'Q' constant.

Since, $C = \frac{Q}{V}$ thus the capacity increases.

This is the principle of the capacitor.

Combination of Capacitors

Parallel combination \rightarrow Number of capacitors are said to be connected in parallel. If the +ve plates of all the capacitors are connected to one point and the -ve plates of all the capacitors are connected to another point and these two points are connected to the two terminals of source maintained at a P.D of (V).



Consider three capacitors having capacities C_1, C_2 & C_3 connected in parallel to a P.D (V).

Let 'q' amount of charge be drawn from the source and q_1, q_2, q_3 be the charges drawn by the capacitors C_1, C_2 & C_3 respectively.

$$\text{Then, } q = q_1 + q_2 + q_3 \quad \text{--- (1)}$$

(According to the law of conservation of charge)

Let C be the net capacity of the combination then, $V = \frac{q}{C}$
 $\Rightarrow q = CV$

Since the P.D across each capacitor is same, therefore;

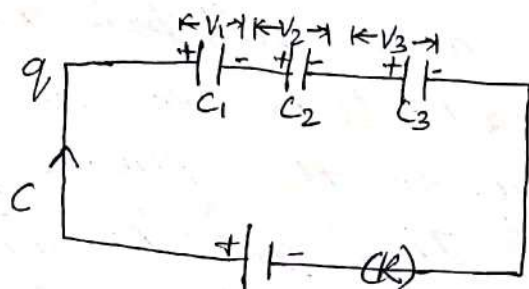
$$V = \frac{q_1}{C_1} = \frac{q_2}{C_2} = \frac{q_3}{C_3}$$

$$\Rightarrow q_1 = VC_1, q_2 = VC_2, q_3 = VC_3$$

substituting in eqn (1) $CV = C_1V + C_2V + C_3V \Rightarrow \boxed{C = C_1 + C_2 + C_3}$

Therefore, when number of capacitors are connected in parallel then the net capacity of the combination is always equal to the sum of their individual capacity.

Series Combination → Numbers of capacitors are said to be connected in series if the +ve plate of the first capacitor is connected to the -ve plate of the next and so on. It leaves the +ve plate of the first capacitor & the -ve plate of the last capacitor as free terminals. These two plates are connected to the two terminals of the source maintained at some potential difference.



Consider three capacitors having capacities C_1, C_2 & C_3 are connected in series to a source maintained at a potential (V).

Let V_1, V_2, V_3 be the potential differences across the capacitors then, $V = V_1 + V_2 + V_3$ — (1)

Let q be the amount of charge drawn from the source 'S' which is same for all the capacitors.

If C be the net capacitors of the combination then, $V = \frac{q}{C}$

Similarly, the P.D across the capacitors $V_1 = \frac{q}{C_1}, V_2 = \frac{q}{C_2}, V_3 = \frac{q}{C_3}$

Substituting in eqn (1)

$$\frac{q}{C} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

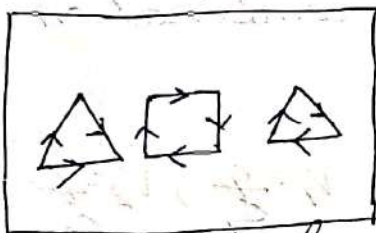
$$\Rightarrow \boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

Therefore, if number of capacitors are connected in series then the reciprocal of the resultant capacity is equal to the sum of the reciprocal of the individual capacities of the capacitors.

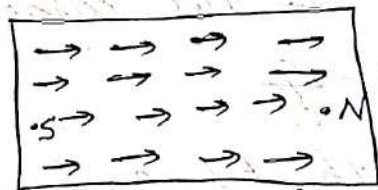
Magnet \rightarrow The substance which has an ability to attract iron like materials.

In an unmagnetised magnetic substance, the molecular magnets are arranged in the form of a closed chain, neutralizing each other's effect.

A magnet consists of large number of molecular magnets. Each molecular magnet has a north pole and a south pole of equal pole strength. When an unmagnetised magnetic substance is under the influence of an electric field or magnetic field then all the molecular magnets are arranged in such a way that their north-poles are directed in one direction called as north pole of the magnet and the south poles are directed in the opposite direction called as the south pole of the magnet. The number of molecular lines (magnets) directed towards a pole of the magnet gives the strength of the pole called as pole strength & is denoted by (m) .



(Unmagnetised magnetic substance)



(Magnetised magnetic substance)

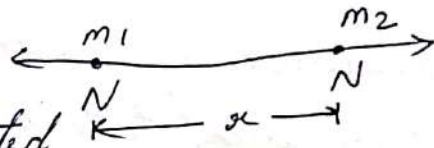
Coulomb's law in magnetism \rightarrow

Coulomb's Law in Magnetostatic

like pole repels each other and unlike poles attract each other. The magnitude of the force of attraction or repulsion between two isolated magnetic poles is given by Coulomb's law.

Statement \rightarrow The magnitude of force of attraction or repulsion between two isolated magnetic poles is directly proportional to the product of their pole strength and inversely proportional to the square of the distance between them.

Q. Consider two isolated magnetic north poles of pole strengths m_1 & m_2 respectively, separated by a distance 'x'. The magnitude of force of repulsion between them is given by



- (i) $F \propto m_1 m_2$
- (ii) $F \propto \frac{1}{x^2}$

Combiningly $F \propto \frac{m_1 m_2}{x^2}$

$$F = \frac{\mu_0}{4\pi} \frac{m_1 m_2}{x^2}$$

where $\frac{\mu_0}{4\pi}$ is a constant,

μ_0 is called permeability of free space.

Unit (magnetic pole) \rightarrow A magnetic pole that, when placed in a vacuum at a distance of one centimeter from an equal and like pole, will repel it with a force of one dyne.

OR

A unit of magnetic pole strength equal to the strength of a magnetic pole that repels an identical pole at a distance of one centimeter with a force of one dyne.

Magnetic field

Magnetic field of a magnet is defined as the space or region surrounding the magnet within which it can influence other magnetic materials.

Magnetic field intensity (H)

The magnetic field intensity at a point within a magnetic field is defined as the amount of force experienced by a unit north pole placed at that point.

The expression for magnetic field intensity can be obtained by substituting $m_1 = 1$ & $m_2 = m$;
in the expression for Coulomb's force

$$F = \frac{\mu_0}{4\pi} \frac{m}{r^2}$$

Magnetic lines of force

Magnetic lines of force is an imaginary path which may be straight or curved along which a unit north pole would move if it were free to do so in a magnetic field. It is a straight line for a single pole and is a curve for multiple poles.

Properties of magnetic lines of force

- ① They are directed away from a north pole & towards a south pole.
- ② They start from a north pole and end at a south pole.
- ③ The tangent drawn at any point on the magnetic lines of force gives the direction of magnetic field intensity at the point.

4) Two magnetic lines of force never intersect. If they do so then at the point of intersection two tangents can be drawn which gives two directions of magnetic field intensity.

5) The number of magnetic lines of force per unit area is directly proportional to the strength of the magnetic field. More concentration of magnetic field lines of force represents a stronger magnetic field.

6) They tend to contract longitudinally this is why two unlike attract each other.

7) They tend to exert lateral pressure this is why two like poles repel each other.

8) A unit north pole produces 4π lines of force.

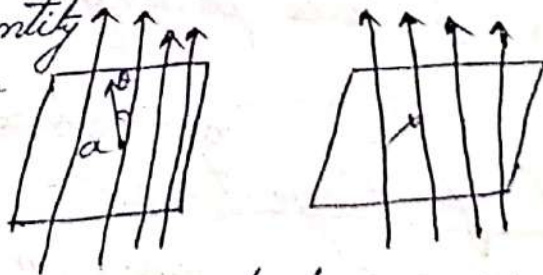
Magnetic flux

Number of magnetic lines of force crossing through an area produces magnetic flux.

Mathematically, it is the dot product of magnetic field induction and area of vector i.e; $\Phi_B = \vec{B} \cdot \vec{a}$
 $= Ba \cos \theta$ - (1)

where θ is the smaller angle between \vec{B} & \vec{a} .

Magnetic flux is a scalar quantity being the dot product of two vector quantities.



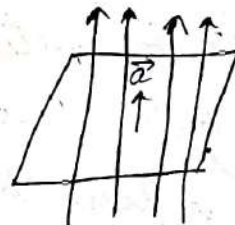
Magnetic flux may also be defined as the product of the magnitude of the magnetic field induction and the component of the area along the direction of magnetic field.

Special cases →

Case ① If $\theta = 0^\circ$, $\cos \theta = 1$

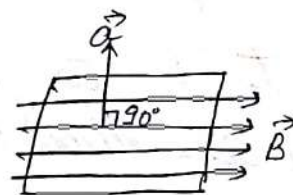
$$\phi_B = Ba \text{ (maximum)}$$

When the magnetic lines of force are crossing the area perpendicularly, the flux is maximum.



Case ② If $\theta = 90^\circ$, $\cos \theta = 0$

$$\phi_B = 0 \text{ (minimum)}$$



i.e; When the magnetic lines of force are parallel to the area, then the flux is zero or minimum.

Again, we know the permeability $\mu = B/H$

$$\Rightarrow B = \mu H$$

$$= \mu a H$$

$$\therefore \text{The } (\phi_B)_{\max} = \mu a H$$

If a coil has 'n' no. of turns then, the maximum flux $(\phi_B)_{\max} = \mu a n H$.

It is to be noted that a magnetic line of force cutting a coil n-times produces equal flux as 'n' no. of lines of force passing a coil once.

Unit of Magnetic flux

- a) In C.G.S → gauss cm^2 or maxwell
- b) In S.I → Tesla m^2 or weber

Relation between weber and Maxwell

$$\begin{aligned} 1 \text{ weber} &= 1 \text{ Tesla} \times 1 \text{ m}^2 \\ &= 10^4 \text{ gauss} \times 10^4 \text{ cm}^2 \\ &= 10^8 \text{ maxwell} \end{aligned}$$

$$\boxed{1 \text{ weber} = 10^8 \text{ Maxwell}}$$

Dimensional formula of magnetic flux

$$\begin{aligned} \Phi_B &= B \cdot a \\ &= [M T^{-2} A^{-1}] [L^2] \\ &= [M L^2 T^{-2} A^{-1}] \end{aligned}$$

Magnetic flux density (B)

It is defined as the sum of the number of magnetic lines of force per unit area of the magnetic field and the number of magnetic lines of force per unit area of the induced magnet.

The number of magnetic lines of force per unit area of the magnetic field is given by H .

A unit pole strength produces 4π lines of force.

Total no. of ^{lines of} force produced by the induced magnet = $4\pi m$.

The no. of lines of force per unit area of the induced magnet = $\frac{4\pi m}{a}$,

According to the statement $B = H + \frac{4\pi m}{a}$

$$\Rightarrow B = H + 4\pi I.$$

UNIT-10 Current Electricity

The rate of flow of charge is called current.

An electric current is the rate of flow of electric charge past a point or region. An electric current is said to exist when there is a net flow of electric charge through a region. In electric current (circuits) this charge is often carried by electrons moving through a wire.

Current is a scalar quantity.

If the charge flows at an uniform rate then current

$$i = \frac{q}{t}$$

For non-uniform flow of charge $i = \frac{dq}{dt}$

The S-I unit of current is Ampere

$$1 \text{ ampere} = \frac{1 \text{ coulomb}}{1 \text{ second}}$$

The current passing through a conductor is said to be 1 ampere if 1 coulomb of charge passes through any cross-section in 1 second.

Ohm's Law

It states that at constant temperature, the current passing through a conductor of uniform cross-sectional area is directly proportional to the potential difference (V) between the two ends of the conductor.

$$V \propto i$$

$$\frac{V}{i} = R = \text{constant}$$

' R ' is known as the resistance of the conductor.

The applications of Ohm's law are as follows:

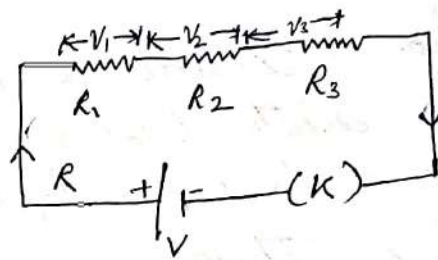
- 1) To determine the voltage, resistance or current of an electric circuit.
- 2) Ohm's law is used to maintain the desired voltage drop across the electronic components.
- 3) Ohm's law is also used in dc ammeter and other dc shunts to divert the current.

Resistance → The obstruction offered to the flow of current is called resistance.

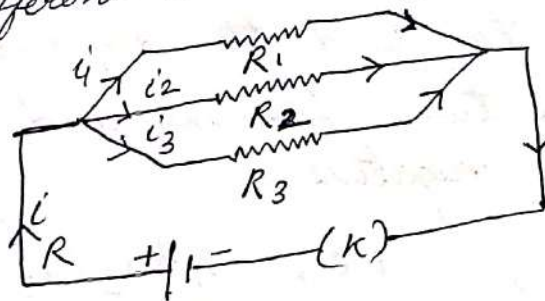
Combination of resistances

Series combination → No. of resistances are said to be connected in series if equal amount of current passes through all of them.

$$R = R_1 + R_2 + R_3$$



Parallel combination → No. of resistances are said to be connected in parallel if different amount of current passes through them.



$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

In series combination, when no. of resistances are connected in series then the net resistance of the combination is equal to the sum of their individual resistances.

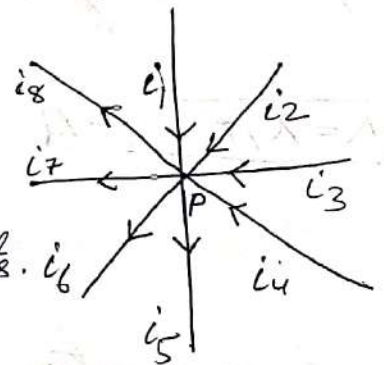
In parallel, when no. of resistances are connected in parallel then the reciprocal of the net resistance is equal to the sum of the reciprocals of the individual resistances.

Kirchoff's law of Electricity

Kirchoff gave two laws on electric circuit

① Kirchoff's first law → It states that the algebraic sum of the current meeting at a junction in a electric circuit is always zero.

Explanation → Consider a point 'P' connected with 5 or 8 wires carrying currents i_1, i_2, i_3, i_4, i_5 are connected in them.



Let i_1, i_2 & i_3 be the incoming currents. i_4 & i_5 are the outgoing currents.

In order to find the algebraic sum the incoming currents are taken positive and the outgoing currents are taken negative.

$$\text{i.e.; } i_1 + i_2 + i_3 - i_4 - i_5 = 0$$

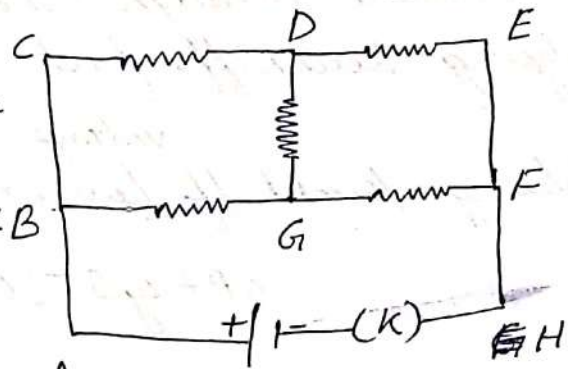
$$\Rightarrow i_1 + i_2 + i_3 = i_4 + i_5$$

i.e.; the total incoming current = total outgoing current and hence charge cannot be charged in an electric circuit or at a junction.

Kirchoff's first law is also known as Kirchoff's current law and is mathematically represented as $\sum i = 0$

② Kirchoff's second / Voltage law \rightarrow It states that in a closed electric circuit the algebraic sum of the emf is equal to the algebraic sum of the product of the current and resistances i.e., $\sum iR = \sum E$

Explanation \rightarrow Consider an electric circuit as shown in the figure. Let R_1, R_2, R_3, R_4 & R_5 be the resistances of the different branches and i_1, i_2, i_3, i_4 & i_5 be the current through them.



Applying Kirchoff's voltage law in the circuit $BCDGB = i_1 R_1 + i_3 R_3 - i_2 R_2 = 0$ — (i)

Similarly, in the circuit $DEFG = i_4 R_4 - i_3 R_3 - i_5 R_5 = 0$ — (ii)

In the circuit $ABGFH = i_2 R_2 + i_5 R_5 = E$ — (iii)

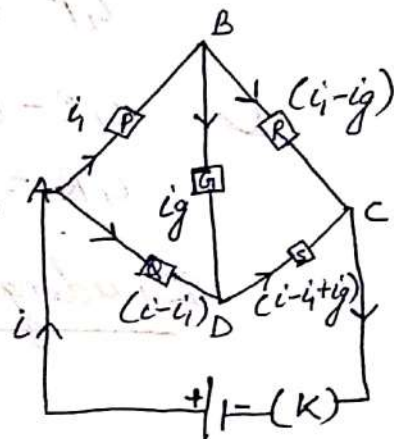
Application of Kirchoff's law

Wheatstone bridge \rightarrow Wheatstone bridge is an electrical arrangement to find the unknown resistance of a wire.

Arrangement

It consists of four wires connected as the four arms of a square ABCD.

P, Q, R & S are the resistances. Three resistances are known and one is unknown.



A galvanometer with resistance 'G' is connected between B & D. A source of e.m.f 'E' is connected between the terminals A & C through a key 'k'.

Calculation

Let i be the current in the circuit.

Let i_g be the current through the galvanometer.

The current through other branches is obtained by applying Kirchhoff's first law.

Applying Kirchhoff's ^{voltage} law in the mesh ABD.

$$iP + i_g G - (i - i_g) Q = 0 \quad \text{--- (I)}$$

Applying KVL in the mesh BCD

$$(i - i_g) R - (i - i_g + i_g) S - i_g G = 0 \quad \text{--- (II)}$$

Now, the known resistances are adjusted in such a way that, the galvanometer shows no deflection i.e.; $i_g = 0$. This is called the balanced condition of the wheatstone bridge.

At balanced condition the equation (I) & (II) reduces to

$$iP - (i - i_g) Q = 0$$

$$iP = (i - i_g) Q \quad \text{--- (III)}$$

$$\text{& } iR - (i - i_g) S = 0$$

$$iR = (i - i_g) S \quad \text{--- (IV)}$$

Dividing eqn (III) by eqn (IV)

$$\frac{iP}{iR} = \frac{(i - i_g) Q}{(i - i_g) S} \Rightarrow \frac{P}{R} = \frac{Q}{S} \quad \text{OR} \quad \boxed{\frac{P}{Q} = \frac{R}{S}}$$

Putting the values of three known resistances the fourth unknown resistance can be calculated.

It is to be noted that if the position of the battery and the galvanometer are interchanged, the Wheatstone bridge still remains balanced.

UNIT - II Electromagnetism & Electromagnetic Induction

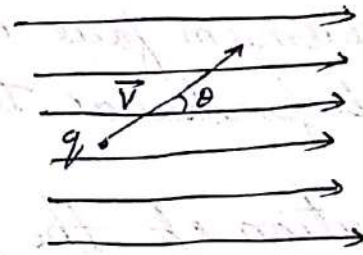
Electromagnetism → Electromagnetism is a branch of Physics that deals with the electromagnetic force that occurs between electrically charged particles. The electromagnetic force is one of the four fundamental forces exhibited by electromagnetic fields such as magnetic fields, electric fields and light. It is the basic reason electrons are bound to the nucleus and responsible for the complete structure of the nucleus.

Electromagnetism is produced when an electrical current flows through a simple conductor such as a length of wire or cable, and as current passes along the whole of the conductor then a magnetic field is created along the whole of the conductor. The small magnetic field created around the conductor has a definite direction with both the North & South poles produced being determined by the direction of the electrical current flowing through the conductor.

Therefore, it is necessary to establish a relationship between current flowing through the conductor and the resultant magnetic field produced around it by the flow of current allowing us to define the relationship that exists between 'Electricity' and 'Magnetism' in the form of Electromagnetism.

Force acting on a current carrying conductor placed in a uniform magnetic field \rightarrow

Consider a charge particle 'q' moving with velocity \vec{v} in a uniform magnetic field \vec{B} making an angle θ with it.



The force experienced by the charge particle is given by

$$\vec{F} = q(\vec{v} \times \vec{B})$$
$$= qvB \sin \theta (\hat{n}) \quad \text{--- (I)}$$

where \hat{n} is the unit vector perpendicular to the plane containing \vec{v} & \vec{B} .

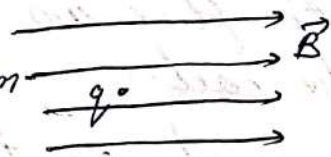
The magnitude of force is given by

$$|\vec{F}| = qvB \sin \theta \quad \text{--- (II)}$$

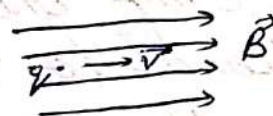
Special Cases

Case-I \rightarrow If $v=0$, then $|\vec{F}|=0$

If a charge is at rest in an uniform magnetic field, it does not experience any force.



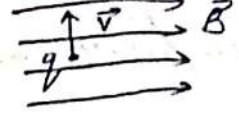
Case-II \rightarrow If $\theta = 0^\circ$ or 180° , $\sin \theta = 0$
 $\therefore |\vec{F}| = 0$



i.e; if a charge is moving parallel to the magnetic field it does not experience any force.

Case-III If $\theta = 90^\circ$, $\sin \theta = 1$

$\therefore |\vec{F}| = qvB$ (maximum)



If a

If a maximum force is experienced by a charge particle moving perpendicular to the magnetic field.

The direction of the force can be obtained by applying cross-product rule or Fleming-left hand rule which may be stated as follows:-

Fleming's Left Hand Rule ->

Stretch first finger, middle finger and thumb of your left hand in mutually perpendicular directions. If the first finger represents the direction of magnetic field, middle finger represents the direction of motion of the charge particle then, the thumb will represent the direction of force experienced by the charge particle.

Faraday's laws of Electromagnetic Induction ->

There are three laws ->

- ① Whenever a coil is linked with a change in magnetic flux an e.m.f and hence, a current is induced in the coil.
- ② The e.m.f exists in the coil so long as it is linked with a change in magnetic flux.
- ③ The e.m.f induced in the coil is directly proportional to the negative rate of change of magnetic flux i.e; $e \propto -\frac{d\phi_B}{dt}$

$\Rightarrow e = -k \frac{d\phi_B}{dt}$

$\therefore k = 1$

$e = \frac{d\phi_B}{dt}$

If a coil has N no. of turns then,

$$e = -N \frac{d\phi_B}{dt}$$

Lenz's Law

It gives the direction of induced e.m.f.

statement \rightarrow It states that the direction of induced e.m.f. is such that it opposes the very cause which produces it.

OR

The direction of induced e.m.f. always opposes the cause which produces it.

Fleming's Right Hand Rule \rightarrow

Stretch the first finger, middle finger & thumb of your left hand in mutually perpendicular directions. If the first finger represents the direction of magnetic field, middle finger represents the direction of current in the conductor then the thumb will represent the direction of force experienced by the conductor.

Comparison between Fleming's Right Hand Rule and Fleming's Left Hand Rule

Fleming's Right Hand Rule

FRHR is applied when current is induced in a conductor due change in magnetic flux around the conducting coil and it is used to find the induced current direction in a conductor the magnetic flux changes around the conductor.

Fleming's Left Hand Rule

FLHR is applied when a current carrying conductor is placed in a magnetic field and used to find out the direction of force on a current carrying conductor in magnetic field.

Laser → A laser is a device that emits light through a process of optical amplification based on the stimulated emission of electromagnetic radiation. The term laser is originated as an acronym for "light amplification by stimulated emission of radiation".

Laser is a device that stimulates atoms or molecules to emit light at particular wavelengths and amplifies that light, typically producing a very narrow beam of radiation. The emission generally covers an extremely limited range of visible infrared or ultraviolet wavelengths. Many different types of lasers have been developed, with highly varied characteristics. Laser is an acronym for light amplification by the stimulated emission of radiation.

Laser Beam → A beam of radiation produced from a laser, used in surgery, communications, weapons systems, printing, recording and various industrial processes.

A laser beam is a device that emits light through a process of optical amplification based on the stimulated emission of electromagnetic radiation.

Principle of laser

Population Inversion → The redistribution of atomic energy levels that takes place in a system so that laser action can occur. Normally, a system of atom is in temperature equilibrium and there are always more atoms in low energy states than in higher ones.

Optical pumping is a process in which light is used to raise (or pump) electrons from a lower energy level in an atom or molecule to a higher one. It is commonly used in laser construction, to pump the active laser medium so as to achieve population inversion. (inversion)