1. Test Continuity of $f(x)$ at $x = 1$

$$f(x) = \begin{cases} \dfrac{x^7 - 1}{x - 1}, & x \neq 1 \\ 7 & x = 1 \end{cases}$$

Ans.

$$f(x) = \begin{cases} \dfrac{x^7 - 1}{x - 1}, & x \neq 1 \\ 7 & x = 1 \end{cases}$$

$$\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{x^7 - 1}{x - 1}$$

$$= \lim_{x \to 1} \frac{x^7 - 1^7}{x - 1} \qquad \left(\because \lim_{x \to a} \frac{x^n - a^n}{x - a} = n a^{n-1} \right)$$

$$= 7 \cdot 1^{7-1}$$

$$= 7 \cdot 1^6 = 7$$

$\therefore \lim_{x \to 1} f(x)$ = exists.

$f(1) = 7$ = definite

$$\lim_{x \to 1} f(x) = f(1) = 7$$

Hence the given function is continuous at $x = 1$.

2. Find the value of K for which $f(x)$ is continuous at $x = 0$.

$$f(x) = \begin{cases} \dfrac{8^x - 4^x - 2^x + 1}{x^2}, & x \neq 0 \\ K, & x = 0 \end{cases}$$

8. Test the continuity of $f(x)$ at
Where, $f(x) = \begin{cases} x^2+1 & \text{if } x<1 \\ 2 & \text{if } x=1 \\ 3x-1 & \text{if } x>1 \end{cases}$

Ans

$\lim_{x \to 1} f(x)$

L.H.L

$= \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} [x^2+1]$

$= \lim_{h \to 0} [(1-h)^2+1]$

$= (1-0)^2+1$

$= 1+1 = 2$

R.H.L. $= \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (3x-1)$

$= \lim_{h \to 0} [3(1+h)-1]$

$= 3(1+0)-1$

$= 3-1$

$= 2$

$\because$ L.H.L = R.H.L. = 2

$\therefore \lim_{x \to 1} f(x)$ exists

$f(1) = 2 =$ finite

$\lim_{x \to 1} f(x) = f(1) = 2$

Hence the ~~above~~ given function is continuous at $x = 1$.

4. Test continuity of the function at $x=1$

Where $f(x) = \begin{cases} \dfrac{x^2-4x+3}{x-1}, & x \neq 1 \\ 2, & x = 1 \end{cases}$

Ans $\lim_{x\to 1} f(x) = \lim_{x\to 1} \dfrac{x^2-4x+3}{x-1}$

$= \lim_{x\to 1} \dfrac{x^2-3x-x+3}{x-1}$

$= \lim_{x\to 1} \dfrac{x(x-3)-1(x-3)}{x-1}$

$= \lim_{x\to 1} \dfrac{\cancel{(x-1)}(x-3)}{\cancel{(x-1)}}$

$= 1-3 = -2$

$f(1) = 2$

$\therefore \lim_{x\to 1} f(x) \neq f(1)$

Hence the given function is discontinuous at $x=1$.

5. Test the continuity of the function at $x=0$

Where $f(x) = \begin{cases} \dfrac{\sin 3x}{\tan^{-1} 7x}, & x \neq 0 \\ 3/7, & x = 0 \end{cases}$

Ans $\lim_{x\to 0} f(x) = \lim_{x\to 0} \dfrac{\sin 3x}{\tan^{-1} 7x}$

$$= \lim_{x\to 0} \frac{\frac{\sin 3x}{3x} \times 3x}{\frac{\tan^{-1} 7x}{7x} \times 7x}$$

$$= \lim_{x\to 0} \frac{\frac{\sin 3x}{3x}}{\frac{\tan^{-1} 7x}{7x}} \; 3/7$$

$$= 3/7 \; \frac{\lim_{x\to 0} \frac{\sin 3x}{3x}}{\lim_{x\to 0} \frac{\tan^{-1} 7x}{7x}}$$

Put 3
When
y→

Put 7
When
t→

$$= 3/7 \; \frac{\lim_{y\to 0} \frac{\sin y}{y}}{\lim_{t\to 0} \frac{\tan^{-1} t}{t}}$$

$$= 3/7 \times \frac{1}{1}$$

$$= 3/7$$

$f(0) = 3/7 =$ finite

$\because \lim_{x\to 0} f(x) = f(0) = 3/7$

Hence the given function is Continuou
at $x = 0$.

1. $\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x}$

$= \lim_{x\to 0} \frac{(\sqrt{x+1}-1)(\sqrt{x+1}+1)}{x(\sqrt{x+1}+1)}$

$= \lim_{x\to 0} \frac{x+1-1}{x(\sqrt{x+1}+1)}$

$= \lim_{x\to 0} \frac{1}{\sqrt{x+1}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}$

2. $\lim_{x\to 0} \frac{\sin ax}{\sin bx} = \lim_{x\to 0} \frac{\frac{\sin ax}{ax} \times ax}{\frac{\sin bx}{bx} \times bx}$

$= \frac{a}{b} \frac{\lim_{x\to 0} \frac{\sin ax}{ax}}{\lim_{x\to 0} \frac{\sin bx}{bx}}$

$= \frac{a}{b} \frac{\lim_{ax\to 0} \frac{\sin ax}{ax}}{\lim_{bx\to 0} \frac{\sin bx}{bx}}$

$= a/b \times \frac{1}{1} = a/b$

3. $\lim_{x\to 0} \frac{\sin 3x}{\sin 2x} = 3/2.$

4. $\lim_{x\to 1} \frac{x^2-\sqrt{x}}{\sqrt{x}-1}$

$$= \lim_{x\to1} \frac{(x^2-\sqrt{x})(x^2+\sqrt{x})(\sqrt{x}+1)}{(\sqrt{x}-1)(x^2+\sqrt{x})(\sqrt{x}+1)}$$

$$= \lim_{x\to1} \frac{(x^4-x)(\sqrt{x}+1)}{(x-1)(x^2+\sqrt{x})}$$

$$\cancel{= \lim_{x\to1} \frac{x^4\sqrt{x}+x^4-x\sqrt{x}-x}{(x-1)(x^2+\sqrt{x})}}$$

$$= \lim_{x\to1} \frac{x(x^3-1)(\sqrt{x}+1)}{(x-1)(x^2+\sqrt{x})}$$

$$= \lim_{x\to1} \frac{x\cancel{(x-1)}(x^2+1+x)(\sqrt{x}+1)}{\cancel{(x-1)}(x^2+\sqrt{x})}$$

$$= \frac{\lim_{x\to1} x \times \lim_{x\to1}(x^2+1+x)\times \lim_{x\to1}(\sqrt{x}+1)}{\lim_{x\to1} x^2 + \lim_{x\to1}\sqrt{x}}$$

$$= \frac{1\times(1^2+1+1)\times(1+1)}{1+1}$$

$$= \frac{1\times 3\times \cancel{2}}{\cancel{2}} = 3$$

5) $\lim_{x\to0} 1-$

$= \lim_{x\to0} 2$

$= 2\lim$

$= 2$ li

$= 2$ l

$= 2\times$

$= \frac{1}{2}$

$= \frac{1}{2}\times$

6) l

$=$ l

5) $\lim_{x \to 0} \frac{1-\cos x}{x^2}$

$= \lim_{x \to 0} \frac{2\sin^2 x/2}{x^2}$

$= 2\lim_{x \to 0} \left(\frac{\sin x/2}{x}\right)^2$

$= 2\lim_{x \to 0} \left(\frac{\sin x/2}{\frac{x}{2} \times 2}\right)^2$

$= 2\lim_{x \to 0} \left(\frac{\sin x/2}{x/2}\right)^2 \times \frac{1}{4}$

$= 2 \times \frac{1}{4}\left(\lim_{x \to 0} \frac{\sin x/2}{x/2}\right)^2$

$= \frac{1}{2}\left(\lim_{\frac{x}{2} \to 0} \frac{\sin x/2}{x/2}\right)^2$

$= \frac{1}{2} \times 1 = \frac{1}{2}$

6) $\lim_{x \to \pi} \frac{\sin x}{\pi - x}$

$= \lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi - x}$ $(\because \sin(\pi - \theta) = \sin\theta)$

Put $\pi - x = t$

When $x \to \pi$ $t \to 0$

$\lim_{t \to 0} \frac{\sin t}{t} = 1$

7. $\lim_{x \to 0} \ln (1+bx)^{1/x}$

$= \ln \lim_{x \to 0} (1+bx)^{1/x}$ $\left(\because \lim_{x \to a} \log_b f \ldots = \log_b \lim_{x \to a} f \ldots \right.$

$= \ln \lim_{x \to 0} (1+bx)^{\frac{1}{bx} \times b}$

$= \ln \left\{ \lim_{x \to 0} (1+bx)^{\frac{1}{bx}} \right\}^b$

$= \ln e^b$

$= b \ln e$

$= b$

8) $\lim_{x \to 0} \frac{\tan 5x}{\tan 7x}$

$= \lim_{x \to 0} \frac{\frac{\tan 5x}{5x} \times 5x}{\frac{\tan 7x}{7x} \times 7x}$

$= \frac{5}{7} \frac{\lim_{5x \to 0} \frac{\tan 5x}{5x}}{\lim_{7x \to 0} \frac{\tan 7x}{7x}}$

$= \frac{5}{7} \times \frac{1}{1} = \frac{5}{7}$

Find a

9. If $\lim_{x \to \alpha} \frac{\tan a(x-\alpha)}{x-\alpha} = \frac{1}{2}$

Ans: $\lim_{x \to \alpha} \frac{\tan a(x-\alpha)}{x-\alpha} = \frac{1}{2}$

$\Rightarrow \lim_{x \to \alpha} \frac{\tan a(x-\alpha)}{a(x-\alpha)} \times a = \frac{1}{2}$

$\Rightarrow a \lim_{x \to \alpha} \frac{\tan a(x-\alpha)}{a(x-\alpha)} = \frac{1}{2}$

put $a(x-\alpha) = t$
when $x \to \alpha$, $t \to 0$

$\therefore a \lim_{t \to 0} \frac{\tan t}{t} = \frac{1}{2}$

$\Rightarrow a \times 1 = \frac{1}{2}$

$\Rightarrow a = \frac{1}{2}.$

10. Find a if $\lim_{x \to 0} \frac{e^{ax} - e^{x}}{x} = 2$

Ans: $\lim_{x \to 0} \frac{e^{ax} - e^{x}}{x} = 2$

$\Rightarrow \lim_{x \to 0} \frac{(e^{ax} - 1) - (e^{x} - 1)}{x} = 2$

$\Rightarrow \lim_{x \to 0} \frac{e^{ax} - 1}{x} - \lim_{x \to 0} \frac{e^{x} - 1}{x} = 2$

$\Rightarrow \lim_{x \to 0} \frac{e^{ax} - 1}{ax} \times a - \log_e e = 2$

$\Rightarrow a \lim_{ax \to 0} \frac{e^{ax} - 1}{ax} - 1 = 2$

$\Rightarrow a \times 1 - 1 = 2$

$\Rightarrow a = 2 + 1 = 3$

11. $\lim_{x \to 2} \frac{\log_e (2x - 3)}{a(x - 2)} = 1$

Find a

Ans: $\Rightarrow \frac{1}{a} \lim_{x \to 2} \frac{\log_e (2x - 3)}{x - 2} = 1$

$\Rightarrow \lim_{x \to 2} \frac{\log_e [1 + (2x - 4)]}{x - 2} = a$

$\Rightarrow \lim_{x \to 2} \frac{\log_e [1 + 2(x - 2)]}{2(x - 2)} \times 2 = a$

$$\Rightarrow 2 \lim_{x \to 2} \frac{\log_e[1 + 2(x-2)]}{2(x-2)} = a$$

Put $2(x-2) = t$

When $x \to 2$, $t \to 0$

$$\Rightarrow 2 \lim_{t \to 0} \frac{\log_e(1+t)}{t} = a$$

$\left(\because \lim_{x \to 0} \frac{\log_e(1+x)}{x} = \log_e e = 1 \right)$

$$\Rightarrow 2 \times 1 = a$$

$$\Rightarrow a = 2$$

12. $$\lim_{x \to 1} \frac{2^{x-1} - 1}{\sqrt{x} - 1}$$

Put $x - 1 = t$

When $x \to 1$, $t \to 0$

$x = t + 1$

$\sqrt{x} = \sqrt{t+1}$

$$= \lim_{t \to 0} \frac{2^t - 1}{\sqrt{t+1} - 1}$$

$$= \lim_{t \to 0} \frac{(2^t - 1)(\sqrt{t+1} + 1)}{(\sqrt{t+1} - 1)(\sqrt{t+1} + 1)}$$

$$= \lim_{t \to 0} \left(\frac{2^t - 1}{t + 1 - 1} \right) \left(\sqrt{t+1} + 1 \right)$$

$$= \lim_{t \to 0} \frac{2^t - 1}{t} \lim_{t \to 0} \left(\sqrt{t+1} + 1 \right)$$

$$= \log_e 2 \times (\sqrt{1} + 1)$$

$2 \log_e 2$