

GOVERNMENT POLYTECHNIC, DEBAGADA

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DEPARTMENT OF MATH AND SCIENCE

MATHEMATICS

Lecture Notes

Classroom Lecture Notes on

Unit-1 TRIGONOMETRY

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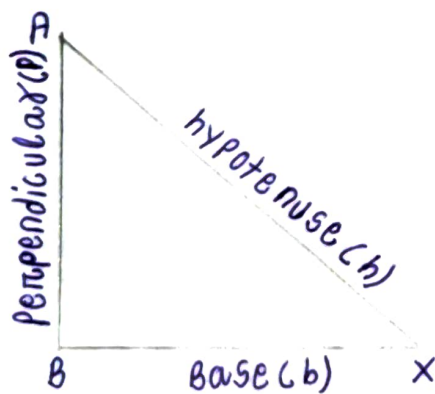
PTGF- Mathematics Faculty

GOVERNMENT POLYTECHNIC, DEBAGADA

Trigonometry

Trigonometry Table

Degree	0°	30°	45°	60°	90°	180°	270°	360°
Radian	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined	0	Not defined	0
Cosec	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	Not defined	-1	Not defined
Sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined	-1	Not defined	1
Cot	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	Not defined	0	Not defined



$$\sin \theta = \frac{p}{h}$$

$$\cos \theta = \frac{b}{h}$$

$$\tan \theta = \frac{p}{b}$$

$$\cot \theta = \frac{b}{p}$$

$$\sec \theta = \frac{h}{b}$$

$$\csc \theta = \frac{h}{p}$$

$$\sin \theta = \frac{1}{\csc \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

Basic Trigonometric Ratios

Complementary Angles

$\sin(90^\circ - \theta) = \cos \theta$	$\sin(90^\circ + \theta) = \cos \theta$
$\cos(90^\circ - \theta) = \sin \theta$	$\cos(90^\circ + \theta) = -\sin \theta$
$\tan(90^\circ - \theta) = \cot \theta$	$\tan(90^\circ + \theta) = -\cot \theta$
$\cot(90^\circ - \theta) = \tan \theta$	$\cot(90^\circ + \theta) = -\tan \theta$
$\sec(90^\circ - \theta) = \operatorname{cosec} \theta$	$\sec(90^\circ + \theta) = -\operatorname{cosec} \theta$
$\operatorname{cosec}(90^\circ - \theta) = \sec \theta$	$\operatorname{cosec}(90^\circ + \theta) = \sec \theta$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Standard Values
Table

$$\sin: 0 \text{ to } 1$$

$$\cos: 1 \text{ to } 0$$

$$\tan 90^\circ = \text{N.D.}!$$

Remember $30^\circ, 45^\circ, 60^\circ!$

Quick Memory Tips

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$\sec^2 \theta - \tan^2 \theta = 1$$

$$\sec^2 \theta = 1 + \tan^2 \theta$$

$$\sec^2 \theta - 1 = \tan^2 \theta$$

$$\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$$

$$\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$$

$$\operatorname{cosec}^2 \theta - 1 = \cot^2 \theta$$

$$\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$$

$$\sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B$$

$$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$$

$$\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B$$

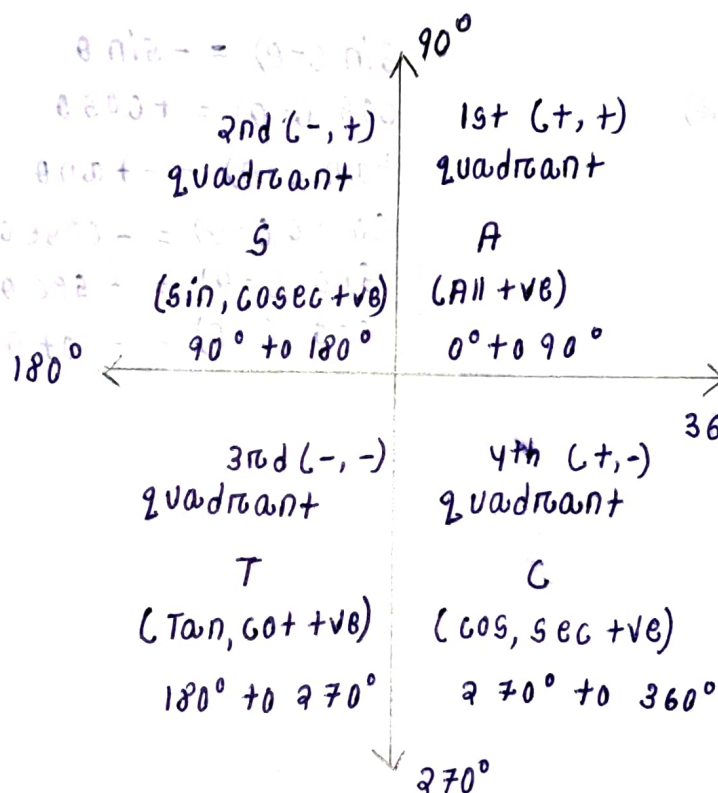
$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$$

$$\begin{aligned}\sin(180-\theta) &= \sin\theta \\ \cos(180-\theta) &= -\cos\theta \\ \tan(180-\theta) &= -\tan\theta \\ \cot(180-\theta) &= -\cot\theta \\ \sec(180-\theta) &= -\sec\theta \\ \operatorname{cosec}(180-\theta) &= \operatorname{cosec}\theta\end{aligned}$$

$$\begin{aligned}\sin(180+\theta) &= -\sin\theta \\ \cos(180+\theta) &= -\cos\theta \\ \tan(180+\theta) &= \tan\theta \\ \cot(180+\theta) &= \cot\theta \\ \sec(180+\theta) &= -\sec\theta \\ \operatorname{cosec}(180+\theta) &= -\operatorname{cosec}\theta\end{aligned}$$

$$\begin{aligned}\sin\theta \cdot \operatorname{cosec}\theta &= 1 \\ \cos\theta \cdot \sec\theta &= 1 \\ \tan\theta \cdot \cot\theta &= 1\end{aligned}$$



H.W

$$\cot(A+B) = ?$$

$$\cot(A-B) = ?$$

ans:
$$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$$

$$\cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

$$\sin 2u = 2 \sin u \cos u$$

$$\sin 2u = \frac{2 \tan u}{1 + \tan^2 u}$$

$$\cos(2u) = \cos^2(u) - \sin^2(u)$$

$$\cos(2u) = 2 \cos^2(u) - 1$$

$$\cos(2u) = 1 - 2 \sin^2(u)$$

$$\cos(2u) = \frac{1 - \tan^2(u)}{1 + \tan^2(u)}$$

$$\tan(2u) = \frac{2 \tan(u)}{1 - \tan^2(u)}$$

$$\cos(2u) = ?$$

1. Basic Form

$$\cos^2(u) - \sin^2(u)$$

2. Cosine only

$$2 \cos^2(u) - 1$$

3. Sine only

$$1 - 2 \sin^2(u)$$

4. Tangent Form

$$\frac{1 - \tan^2(u)}{1 + \tan^2(u)}$$

$$\sin^2 u + \cos^2 u = 1$$

$$\Rightarrow \sin^2 u = 1 - \cos^2 u$$

$$= \cos^2 u - (1 - \cos^2 u)$$

$$= \cos^2 u - 1 + \cos^2 u$$

$$2 \cos^2 u - 1 \quad (\cos)$$

$$\sin^2 u + \cos^2 u = 1$$

$$\Rightarrow \cos^2 u = 1 - \sin^2 u$$

$$= \sin^2 u - (1 - \sin^2 u)$$

$$= \sin^2 u - 1 + \sin^2 u$$

$$2 \sin^2 u - 1 \quad (\sin)$$

$$\sin(-\theta) = -\sin \theta$$

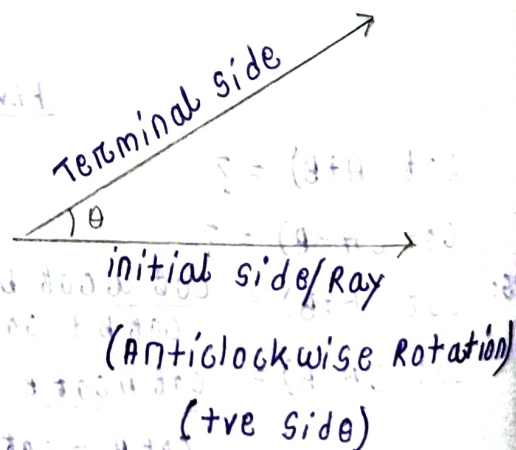
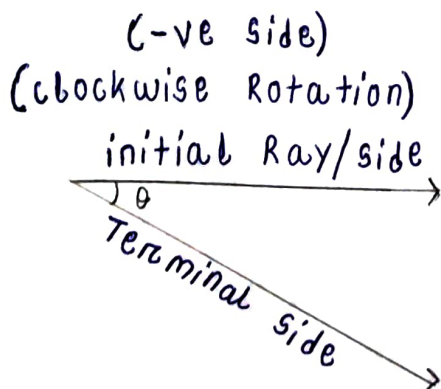
$$\cos(-\theta) = +\cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\operatorname{cosec}(-\theta) = -\operatorname{cosec} \theta$$

$$\sec(-\theta) = +\sec \theta$$

$$\cot(-\theta) = -\cot \theta$$



What is Terminal side?

- Angle is a measure of rotation of a given ray about its initial point.
- The original ray is called initial side and the final position of the ray after rotation is called terminal side of the angle.

Degree $\rightarrow$ Radian

$$\text{degree} \times \frac{\pi}{180^\circ}$$

Radian $\rightarrow$ Degree

$$\text{Radian} \times \frac{180^\circ}{\pi}$$

Q Convert $30^\circ, 45^\circ, 60^\circ, 90^\circ, 100^\circ$ in Radian

ans: $30^\circ = 30^\circ \times \frac{\pi}{180^\circ}$

$$= \frac{\pi}{6}$$

$$45^\circ = 45^\circ \times \frac{\pi}{180^\circ}$$

$$= \frac{\pi}{4}$$

$$60^\circ = 60^\circ \times \frac{\pi}{180^\circ}$$

$$= \frac{\pi}{3}$$

$$90^\circ = 90^\circ \times \frac{\pi}{180^\circ}$$

$$= \frac{\pi}{2}$$

$$100^\circ = 100^\circ \times \frac{\pi}{180^\circ}$$

$$= \frac{10\pi}{18}$$

$$= \frac{5\pi}{9}$$

Q Convert $\frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{6}$ in degree

$$\frac{\pi}{3} = \frac{\pi}{3} \times \frac{180^\circ}{\pi}$$

$$= 60^\circ$$

$$\frac{\pi}{4} = \frac{\pi}{4} \times \frac{180^\circ}{\pi}$$

$$= 45^\circ$$

$$\frac{\pi}{6} = \frac{\pi}{6} \times \frac{180^\circ}{\pi}$$

$$= 30^\circ$$

Grade

It is defined as the one-hundredth of right angle i.e., There are 100 gradient in 90° .

Relation between degree, grade, radian

$$\frac{D}{90} = \frac{G}{100} = \frac{R}{\pi}$$

$$X^G = \left(\frac{90}{10} \right)^D$$

$$X^G = \left(\frac{\pi u}{200} \right)^R$$

$$X^D = \left(\frac{10u}{9} \right)^G$$

$$X^D = \left(\frac{\pi}{180} \right)^R$$

$$X^R = \left(\frac{180u}{\pi} \right)^D$$

$$X^R = \left(\frac{200u}{\pi} \right)^G$$

Sexagesimal system / English system

$$1^\circ = 60 \text{ min}, 60'$$

$$1' = 60 \text{ sec}, 60''$$

Centesimal System

$$90^\circ = 100 \text{ Grade}$$

$$1 \text{ Grade} = 100 \text{ min}$$

$$1 \text{ min} = 100 \text{ sec}$$

Q $50^G = \text{--- Radian}$

$$X^G = \left(\frac{\pi \times 100}{200} \right)^R$$

$$50^G = \left(\frac{\pi \times 50}{200} \right)^R$$

$$50^G = \frac{50\pi}{200}$$

$$= \frac{\pi}{4}$$

Q $100^G = \text{--- Degree}$

$$X^G = \left(\frac{90}{100} \right)^D$$

$$100^G = \left(\frac{90 \times 100}{100} \right)^D$$

$$100^G = \left(\frac{9000}{100} \right)^D$$

$$= \frac{90}{1}$$

$$= 90^\circ$$

Q Degree to Grade

$$30^\circ, 40^\circ, 60^\circ, 75^\circ, 90^\circ$$

Q Grade to degree

$$30^G, 40^G, 60^G, 75^G, 90^G$$

Q Grade to Radian

$$30^G, 40^G, 60^G$$

Q Radian to Grade

$$\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{5\pi}{3}$$

Answer

Q Degree to Radian

$$X^D = \left(\frac{\pi 16}{180} \right)^R$$

$$\begin{aligned} 30^\circ &= \left(\frac{\pi 30^\circ}{180} \right)^R \\ &= \frac{2\pi}{18} \\ &= \frac{\pi}{6} \end{aligned}$$

$$\begin{aligned} 40^\circ &= \left(\frac{\pi 40^\circ}{180} \right)^R \\ &= \frac{4\pi}{18} \\ &= \frac{2\pi}{9} \end{aligned}$$

$$\begin{aligned} 60^\circ &= \left(\frac{\pi 60^\circ}{180} \right)^R \\ &= \frac{6\pi}{18} \\ &= \frac{\pi}{3} \end{aligned}$$

$$\begin{aligned} 75^\circ &= \left(\frac{\pi 75^\circ}{180} \right)^R \\ &= \frac{75\pi}{180} \\ &= \frac{5\pi}{12} \end{aligned}$$

$$\begin{aligned} 90^\circ &= \left(\frac{\pi 90^\circ}{180} \right)^R \\ &= \frac{9\pi}{18} \\ &= \frac{\pi}{2} \end{aligned}$$

Q Grade to degree

$$X^G = \left(\frac{90}{10} \right)^D$$

$$\begin{aligned} 30^G &= \left(\frac{90 \times 30}{10} \right)^D \\ &= \left(\frac{270}{10} \right) \\ &= 27^\circ \end{aligned}$$

$$\begin{aligned} 40^G &= \left(\frac{90 \times 40}{10} \right)^D \\ &= \left(\frac{360}{10} \right)^D \\ &= 36^\circ \end{aligned}$$

$$\begin{aligned} 75^G &= \left(\frac{90 \times 75}{10} \right)^D \\ &= \left(\frac{675}{10} \right) \\ &= 67.5^\circ \end{aligned}$$

$$\begin{aligned} 90^G &= \left(\frac{90 \times 90}{10} \right)^D \\ &= \left(\frac{810}{10} \right) \\ &= 81^\circ \end{aligned}$$

$$\begin{aligned} 60^G &= \left(\frac{90 \times 60}{10} \right)^D \\ &= \left(\frac{540}{10} \right) \\ &= 54^\circ \end{aligned}$$

Q Grade to Radian

$$X^G = \left(\frac{\pi 16}{200} \right)^R$$

$$\begin{aligned} 30^G &= \left(30 \times \frac{\pi}{200} \right)^R \\ &= \frac{3\pi}{20} \end{aligned}$$

$$\begin{aligned} 40^G &= \left(40 \times \frac{\pi}{200} \right)^R \\ &= \frac{\pi}{5} \end{aligned}$$

$$\begin{aligned} 60^G &= \left(60 \times \frac{\pi}{200} \right)^R \\ &= \frac{3\pi}{10} \end{aligned}$$

Q Radian to Grade

$$X^R = \left(\frac{200\pi}{\pi} \right)^G$$

$$\frac{\pi}{4}^R = \frac{\pi}{4} \times \frac{200}{\pi}$$

$$= 50^G$$

$$\frac{\pi}{6}^R = \frac{\pi}{6} \times \frac{200}{\pi}$$

$$= \frac{100}{3}$$

$$\frac{\pi}{9}^R = \frac{\pi}{9} \times \frac{200}{\pi}$$

$$= \frac{200}{9}$$

$$\frac{5\pi}{3}^R = \frac{5\pi}{3} \times \frac{200}{\pi}$$

$$= \frac{1000}{3}$$

Q Degree to Grade

$$X^D = \left(\frac{100}{9} \right)^G$$

$$30^\circ = 30 \times \frac{10}{9}$$

$$= \frac{300}{9}$$

$$= 33.33^G$$

$$40^\circ = 40 \times \frac{10}{9}$$

$$= \frac{400}{9}$$

$$= 44.44^G$$

$$60^\circ = 60 \times \frac{10}{9}$$

$$= \frac{600}{9}$$

$$= 66.67^G$$

$$75^\circ = 75 \times \frac{10}{9}$$

$$= \frac{750}{9}$$

$$= 83.33^G$$

$$90^\circ = 90 \times \frac{10}{9}$$

$$= \frac{900}{9}$$

$$= 100.00 \text{ grade}$$

CW

$$1^R = \underline{57} \text{ degree}$$

Ailed Angle

Two angle is said to be an ailed angle if their sum or difference is the multiple of 0° or 90° .

CW

$$\frac{5\pi}{9} = ?$$

$$\frac{5}{9} \times \frac{22}{7} = \frac{110}{63}$$

$$\frac{110}{63} \approx 1.746$$

$$\sin 1 > \sin 1^\circ$$

$$a) =$$

$$b) <$$

$$c) >$$

d) None ($\because 1^\circ = \frac{\pi}{180}$ which is greater than $1'$)

Rule 1

If the angle θ is an acute angle is increase or decrease by a multiple of 360° then $\pm$ -ratio remain same.

Rule 2

$\pm$ -ratio of an angle $-\theta, \pi \pm \theta, 2\pi \pm \theta$ remain same

Rule 3

$\pm$ -ratio of angle $\frac{\pi}{2} \pm \theta, \frac{3\pi}{2} \pm \theta, \frac{5\pi}{2} \pm \theta$ change as $\sin \leftrightarrow \cos$

$$\tan \leftrightarrow \cot$$

$$\sec \leftrightarrow \csc$$

CW

$$\sin\left(\frac{3\pi}{2} + \theta\right) = -\cos \theta$$

$$\sin\left(\frac{3\pi}{2} - \theta\right) = -\cos \theta$$

HW

Write $\pm$ -ratio of $\sin\left(\frac{\pi}{2} - \theta\right), \cos\left(\frac{\pi}{2} - \theta\right), \tan\left(\frac{\pi}{2} - \theta\right), \cot\left(\frac{\pi}{2} - \theta\right), \sec\left(\frac{\pi}{2} - \theta\right), \csc\left(\frac{\pi}{2} - \theta\right), \sin\left(\frac{\pi}{2} + \theta\right), \cos\left(\frac{\pi}{2} + \theta\right), \tan\left(\frac{\pi}{2} + \theta\right), \cot\left(\frac{\pi}{2} + \theta\right), \sec\left(\frac{\pi}{2} + \theta\right), \csc\left(\frac{\pi}{2} + \theta\right)$

Answer

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$$

$$\cot\left(\frac{\pi}{2} - \theta\right) = \tan \theta$$

$$\sec\left(\frac{\pi}{2} - \theta\right) = \csc \theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec \theta$$

$$\sin\left(\frac{\pi}{2} + \theta\right) = \cos \theta$$

$$\cos\left(\frac{\pi}{2} + \theta\right) = -\sin \theta$$

$$\tan\left(\frac{\pi}{2} + \theta\right) = -\cot \theta$$

$$\cot\left(\frac{\pi}{2} + \theta\right) = -\tan \theta$$

$$\sec\left(\frac{\pi}{2} + \theta\right) = -\csc \theta$$

$$\csc\left(\frac{\pi}{2} + \theta\right) = -\sec \theta$$

CW

$$\sin\left(\frac{3\pi}{2} + \theta\right) = -\cos \theta$$

$$\sin(4\pi + \theta) = \sin \theta$$

$$\csc\left(\frac{3\pi}{2} + \theta\right) = -\sec \theta$$

$$\sin(A+B) \sin(A-B) = \sin^2 A \cdot \sin^2 B = \cos^2 B \cdot \cos^2 A$$

$$\cos(A+B) \cos(A-B) = \cos^2 A \cdot \sin^2 B = \cos^2 B \cdot \sin^2 A$$

Q Evaluate $\sin(75^\circ)$

$$\sin(45^\circ + 30^\circ) = \sin \frac{1}{\sqrt{2}} \cdot \cos \frac{\sqrt{3}}{2} + \cos \frac{1}{\sqrt{3}} \cdot \sin \frac{1}{2}$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2} + \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Q $\tan(75^\circ)$

$$\tan(45^\circ) = (45^\circ - 30^\circ)$$

$$= \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \times \frac{1}{\sqrt{3}}}$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}} \cdot \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

Q $\tan(57^\circ)$

$\tan(45^\circ + 12^\circ)$

$$= \frac{\tan 45^\circ + \tan 12^\circ}{1 - \tan 45^\circ \cdot \tan 12^\circ}$$

$$= \frac{1 + \tan 12^\circ}{1 - 1 \times \tan 12^\circ}$$

$$= \frac{1 + \tan 12^\circ}{1 - \tan 12^\circ}$$

$$= \frac{1 + \frac{\sin 12^\circ}{\cos 12^\circ}}{1 - \frac{\sin 12^\circ}{\cos 12^\circ}}$$

$$= \frac{\cos 12^\circ + \sin 12^\circ}{\cos 12^\circ - \sin 12^\circ}$$

$$= \frac{\cos 12^\circ + \sin 12^\circ}{\cos 12^\circ - \sin 12^\circ}$$

$$= \frac{\cos 12^\circ + \sin 12^\circ}{\cos 12^\circ - \sin 12^\circ}$$

$$= \frac{\cos 12^\circ + \sin 12^\circ}{\cos 12^\circ - \sin 12^\circ}$$

HW

$\cos(75^\circ)$

$\tan(75^\circ)$

$\cot(75^\circ)$

$\sec(75^\circ)$

$\operatorname{cosec}(75^\circ)$

$\sin(15^\circ)$

$\cos(15^\circ)$

$\operatorname{cosec}(15^\circ)$

$\sec(15^\circ)$

$\cot(15^\circ)$

$\sin(57^\circ)$

$\cos(57^\circ)$

$\cot(57^\circ)$

$\sec(57^\circ)$

$\operatorname{cosec}(57^\circ)$

$\tan 50^\circ = \tan 40^\circ + 2 \tan 10^\circ$

Convert $40^\circ 20'$ into radian

Find the $\pm$ -ratio of the following:

$\sin(480^\circ)$

$\tan(\pi + \theta)$

$\sec\left(\frac{3\pi}{2} - \theta\right)$

Answer

Q $\cos(75^\circ) = ?$

$\cos(45^\circ + 30^\circ) = 75^\circ$

$\cos(A+B) = \cos A \cdot \cos B - \sin A \sin B$

$= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

Q $\tan(75^\circ) = ?$

$\tan 75^\circ = \tan(45^\circ + 30^\circ)$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$

$= \frac{\sqrt{3}+1}{\sqrt{3}-1}$

Q $\cot(75^\circ) = ?$

$\cot(45^\circ + 30^\circ) = \cot(75^\circ)$

$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$

$= \frac{\cot 45^\circ \cot 30^\circ - 1}{\cot 45^\circ + \cot 30^\circ}$

$\cot 45^\circ = 1, \cot 30^\circ = \sqrt{3}$

$= \frac{(1)(\sqrt{3}) - 1}{1 + \sqrt{3}}$

$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$

Q $\sec(75^\circ)$

$$75^\circ = 45^\circ + 30^\circ$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos 75^\circ = \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

$$\sec 75^\circ = \frac{2\sqrt{2}}{\sqrt{3} - 1}$$

Q $\operatorname{cosec}(75^\circ)$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$75^\circ = 45^\circ + 30^\circ$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin 75^\circ = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

$$\operatorname{cosec} 75^\circ = \frac{2\sqrt{2}}{\sqrt{3} + 1}$$

Q $\sin(15^\circ)$

$$15^\circ = 45^\circ - 30^\circ$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\sin 15^\circ = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

Q $\cos(15^\circ)$

$$15^\circ = 45^\circ - 30^\circ$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\cos 15^\circ = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Q $\operatorname{cosec}(15^\circ)$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\sin 15^\circ = (45^\circ - 30^\circ)$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\sin(45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\operatorname{cosec} 15^\circ = \frac{2\sqrt{2}}{\sqrt{3}-1}$$

Q $\sec(15^\circ)$

$$\sec 15^\circ = \frac{1}{\cos 15^\circ}$$

$$\cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$\sec 15^\circ = \frac{2\sqrt{2}}{\sqrt{3}+1}$$

Q $\cot(15^\circ)$

$$\cot 15^\circ = \frac{1}{\tan 15^\circ}$$

$$\tan(45^\circ - 30^\circ) = \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

$$\tan 15^\circ = 2 - \sqrt{3}$$

$$\cot 15^\circ = \frac{1}{2 - \sqrt{3}}$$

Q $\sin(57^\circ)$

$$\sin 57^\circ = \sin(45^\circ + 12^\circ)$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin 57^\circ = \sin 45^\circ \cos 12^\circ + \cos 45^\circ \sin 12^\circ$$

$$\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\sin 57^\circ = \frac{\cos 12^\circ + \sin 12^\circ}{\sqrt{2}}$$

Q $\cos(57^\circ)$

$$\cos 57^\circ = \cos(45^\circ + 12^\circ)$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos 57^\circ = \cos 45^\circ \cos 12^\circ - \sin 45^\circ \sin 12^\circ$$

$$\cos 45^\circ = \sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\cos 57^\circ = \frac{\cos 12^\circ - \sin 12^\circ}{\sqrt{2}}$$

Q $\cot(57^\circ)$

$$57^\circ = 45^\circ + 12^\circ$$

$$\cot 57^\circ = \frac{\cot 45^\circ \cot 30^\circ - 1}{\cot A + \cot B}$$

$$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$$

$$\cot 75^\circ = \frac{\cot 45^\circ \cot 30^\circ - 1}{\cot 45^\circ + \cot 30^\circ}$$

$$\cot 45^\circ = 1, \cot 30^\circ = \sqrt{3}$$

$$= \frac{(1)(\sqrt{3}) - 1}{1 + \sqrt{3}}$$

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

Q $\sec(57^\circ)$

$$57^\circ = 45^\circ + 12^\circ$$

$$\sec 57^\circ = \frac{1}{\cos 57^\circ}$$

$$\cos 57^\circ = ?$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos 57^\circ = \cos(45^\circ + 12^\circ)$$

$$= \cos 45^\circ \cos 12^\circ - \sin 45^\circ \sin 12^\circ$$

$$= \frac{1}{\sqrt{2}} (\cos 12^\circ - \sin 12^\circ)$$

$$\cos 12^\circ \approx 0.9781$$

$$\sin 12^\circ \approx 0.2079$$

$$\cos 57^\circ \approx 0.5446$$

$$\sec 57^\circ = \frac{1}{0.5446}$$

$$\sec 57^\circ \approx 1.836$$

Q $\operatorname{cosec}(57^\circ)$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$57^\circ = 45^\circ + 12^\circ$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin 57^\circ = \sin(45^\circ + 12^\circ)$$

$$= \sin 45^\circ \cos 12^\circ + \cos 45^\circ \sin 12^\circ$$

$$= \frac{1}{\sqrt{2}} \cos 12^\circ + \frac{1}{\sqrt{2}} \sin 12^\circ$$

$$= \frac{1}{\sqrt{2}} (0.9781) + \frac{1}{\sqrt{2}} (0.2079)$$

$$= \frac{1.1860}{1.4142}$$

$$\sin 57^\circ \approx 0.8387$$

$$\operatorname{cosec} 57^\circ = \frac{1}{0.8387}$$

Q $\tan 50^\circ = \tan 40^\circ + 2 \tan 10^\circ$

$$50^\circ = 40^\circ + 10^\circ$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan 50^\circ = \frac{\tan 40^\circ + \tan 10^\circ}{1 - \tan 40^\circ \tan 10^\circ}$$

$$\tan 40^\circ \tan 10^\circ = \tan 40^\circ \tan(50^\circ - 40^\circ)$$

$$\tan 40^\circ + 2 \tan 10^\circ = \tan 50^\circ$$

$$\tan 40^\circ + 2 \tan 10^\circ$$

$$= 0.8391 + 2(0.1763)$$

$$= 0.8391 + 0.3526$$

$$= 1.1917$$

$$\tan 50^\circ \approx 1.1918$$

Q Convert $40^\circ 20'$ into radian

ans: Convert minutes into degrees

$$1^\circ = 60'$$

$$20' = \frac{20^\circ}{60} = \frac{1^\circ}{3}$$

$$40^\circ 20' = 40^\circ + \frac{1^\circ}{3}$$

$$= \frac{121^\circ}{3}$$

Convert degree into radians

$$180^\circ = \pi \text{ radians}$$

$$\frac{121^\circ}{3} \times \frac{\pi}{180^\circ}$$

$$= \frac{121\pi}{540}$$

$$40^\circ 20' = \frac{121\pi}{540} \text{ radians}$$

Q T-ratio of $\sin(480^\circ)$

$$480^\circ = 360^\circ + 120^\circ$$

$$\sin 480^\circ = \sin(360^\circ + 120^\circ)$$

$$\sin(360^\circ + \theta) = \sin \theta$$

$$\sin 480^\circ = \sin 120^\circ$$

$$120^\circ = 180^\circ - 60^\circ$$

$$\sin(180^\circ - \theta) = \sin \theta$$

$$\sin 120^\circ = \sin 60^\circ$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\sin 480^\circ = \frac{\sqrt{3}}{2}$$

we can also find by putting $\left(\frac{5\pi}{2} + \theta\right)$, $(450^\circ + 30^\circ)$

Q T-ratio of $\tan(\pi + \theta)$

$$\pi = 180^\circ$$

$$\tan(\pi + \theta) = \tan(180^\circ + \theta)$$

$$\tan(180^\circ + \theta) = \tan \theta$$

$$\tan(\pi + \theta) = \tan \theta$$

Q T-ratio of $\sec\left(\frac{3\pi}{2} - \theta\right)$

Convert $\frac{3\pi}{2}$ into degree

$$\frac{3\pi}{2} = 270^\circ$$

$$\sec\left(\frac{3\pi}{2} - \theta\right) = \sec(270^\circ - \theta)$$

$$\cos(270^\circ - \theta) = -\sin \theta$$

$$\sec u = \frac{1}{\cos u}$$

$$\sec(270^\circ - \theta) = \frac{1}{-\sin \theta}$$

$$\frac{1}{\sin \theta} = \operatorname{cosec} \theta$$

$$\sec\left(\frac{3\pi}{2} - \theta\right) = -\operatorname{cosec} \theta$$

$$\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$$

$$\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\cos C - \cos D = -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\text{or } 2 \sin \frac{D+C}{2} \cdot \sin \frac{D-C}{2}$$

Q Solve $\sin 65^\circ + \cos 65^\circ$

$$\sin 65^\circ + \cos (90^\circ - 25^\circ)$$

$$= \sin 65^\circ + \sin 25^\circ$$

$$= 2 \sin \frac{65^\circ + 25^\circ}{2} \cos \frac{65^\circ - 25^\circ}{2}$$

$$= 2 \sin 45^\circ \cdot \cos 20^\circ$$

$$= 2 \times \frac{1}{\sqrt{2}} \cdot \cos 20^\circ$$

$$= \sqrt{2} \cos 20^\circ$$

Q If $\sin \theta = \frac{3}{5}$ then find $\cos \theta$, $\tan \theta$, $\cot \theta$, $\sec \theta$, $\operatorname{cosec} \theta$

$$\cos \theta = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$= \frac{4}{5}$$

$$\tan \theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$= \frac{3}{4}$$

$$\cot \theta = \frac{\text{Base}}{\text{Perpendicular}}$$

$$= \frac{4}{3}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$= \frac{1}{4/5}$$

$$= \frac{5}{4}$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$= \frac{1}{3/5}$$

$$= \frac{5}{3}$$

HW

1. Prove that $\cot 2\theta + \tan \theta = \operatorname{cosec} 2\theta$
2. Prove that $\frac{\sin(A-B)}{\cos A \cos B} + \frac{\sin(B-C)}{\cos B \cos C} + \frac{\sin(C-A)}{\cos C \cos A} = 0$
3. Find the value of $\tan 135^\circ + \cot 135^\circ$, $\sec 150^\circ + \operatorname{cosec} 15^\circ$

Answer

$$3(i) \tan 135^\circ + \cot 135^\circ$$

$$\tan 135^\circ = \tan(180^\circ - 45^\circ)$$

$$= -\tan 45^\circ = -1$$

$$\cot 135^\circ = -1$$

$$\tan 135^\circ + \cot 135^\circ$$

$$= -1 + (-1)$$

$$= -2$$

$$3(ii) \sec 150^\circ + \operatorname{cosec} 15^\circ$$

$$\sec 150^\circ = \frac{1}{\cos 150^\circ}$$

$$= \cos 150^\circ = \cos(180^\circ - 30^\circ) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$$

$$= \sec 150^\circ = \frac{-2}{\sqrt{3}}$$

$$= \operatorname{cosec} 15^\circ = \frac{1}{\sin 15^\circ}$$

$$\sin 15^\circ = \sin(45^\circ - 30^\circ)$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{6} - \sqrt{3}}{4}$$

$$\operatorname{cosec} 15^\circ = \frac{4}{\sqrt{6} - \sqrt{3}}$$

$$= \frac{4(\sqrt{6} + \sqrt{3})}{6 - 3}$$

$$= \sqrt{6} + \sqrt{2}$$

$$\sec 150^\circ + \operatorname{cosec} 15^\circ$$

$$= -\frac{2}{\sqrt{3}} + \sqrt{6} + \sqrt{2}$$

$$= \sqrt{6} + \sqrt{2} - \frac{2\sqrt{3}}{3}$$

$$7. \frac{\sin(A-B)}{\cos A \cos B} + \frac{\sin(B-C)}{\cos B \cos C} + \frac{\sin(C-A)}{\cos C \cos A} = 0$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\frac{\sin(A-B)}{\cos A \cos B} = \frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B}$$

$$= \tan A - \tan B$$

$$\frac{\sin(B-C)}{\cos B \cos C} = \tan B - \tan C$$

$$\frac{\sin(C-A)}{\cos C \cos A} = \tan C - \tan A$$

$$(\tan A - \tan B) + (\tan B - \tan C) + (\tan C - \tan A)$$

$$= \tan A - \tan A + \tan B - \tan B + \tan C - \tan C$$

$$= 0$$

Hence, proved

$$\tan 50^\circ = \tan(40^\circ + 10^\circ)$$

$$\Rightarrow \tan 50^\circ = \frac{\tan 40^\circ + \tan 10^\circ}{1 - \tan 40^\circ \tan 10^\circ}$$

$$\Rightarrow \tan 50^\circ - \tan 50^\circ \cdot \tan 40^\circ \tan 10^\circ = \tan 40^\circ + \tan 10^\circ$$

$$\Rightarrow \tan 50^\circ - \tan(90 - 40) \tan 40^\circ \tan 10^\circ = \tan 40^\circ + \tan 10^\circ \quad [\because \text{cross multiplication}]$$

$$\Rightarrow \tan 50^\circ - \cot 40^\circ \tan 40^\circ \tan 10^\circ = \tan 40^\circ + \tan 10^\circ$$

$$\Rightarrow \tan 50^\circ - \tan 10^\circ = \tan 40^\circ + \tan 10^\circ$$

$$[\cot 40^\circ \times \tan 40^\circ = 1]$$

$$\Rightarrow \tan 50^\circ = 2 \tan 10^\circ + \tan 40^\circ$$

$$\sin(A+B) + \sin(A-B) = 2 \sin A \cos B$$

$$\sin(A+B) - \sin(A-B) = 2 \cos A \sin B$$

$$\cos(A+B) + \cos(A-B) = 2 \cos A \cos B$$

$$\cos(A+B) - \cos(A-B) = -2 \sin A \sin B$$

$$\text{or } \cos(A-B) - \cos(A+B) = 2 \sin A \cos B$$

HW

1. $\cot 2\theta + \tan \theta = \operatorname{cosec} 2\theta$
2. Table $30^\circ, 45^\circ, 60^\circ, 90^\circ, 120^\circ, 135^\circ, 150^\circ, 180^\circ, 270^\circ, 360^\circ$
3. $\sec 150^\circ + \operatorname{cosec} 150^\circ$

Answer

1. Prove $\cot 2\theta + \tan \theta = \operatorname{cosec} 2\theta$

LHS:

$$\cot \theta + \tan \theta$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}, \quad \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$= \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta}$$

$$= \frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cos \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$= \frac{1}{\sin \theta \cos \theta}$$

This is not equal to $\operatorname{cosec} 2\theta$ in general

$$\cot 2\theta + \tan \theta \neq \operatorname{cosec} 2\theta$$

3. $\sec 150^\circ + \operatorname{cosec} 150^\circ$

$\sec 150^\circ$

$150^\circ = 180^\circ - 30^\circ$

$\cos(180^\circ - \theta) = -\cos \theta$

$\cos 150^\circ = -\cos 30^\circ$

$= -\frac{\sqrt{3}}{2}$

$\sec 150^\circ = \frac{1}{\cos 150^\circ}$

$= -\frac{2}{\sqrt{3}}$

$\operatorname{cosec} 150^\circ$

$\sin 150^\circ = \sin(180^\circ - 30^\circ)$

$= \sin 30^\circ$

$= \frac{1}{2}$

$\operatorname{cosec} 150^\circ = \frac{1}{\sin 150^\circ} = 2$

$\sec 150^\circ + \operatorname{cosec} 150^\circ$

$= -\frac{2}{\sqrt{3}} + 2$

$= 2 - \frac{2}{\sqrt{3}}$

$\frac{2(\sqrt{3} - 1)}{\sqrt{3}}$

2.

Degree	120°	135°	150°
Radian	2.094	$3\pi/4$	$5\pi/6$
sin	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$
cos	$-1/2$	$-1/\sqrt{2}$	$-\sqrt{3}/2$
tan	$-\sqrt{3}$	-1	$-1/\sqrt{3}$
cot	$-1/\sqrt{3}$	-1	$-\sqrt{3}$
sec	-2	$-\sqrt{2}$	$-2/\sqrt{3}$
cosec	$2/\sqrt{3}$	$\sqrt{2}$	2

CW

$$1. \sin 10^\circ \cdot \sin 30^\circ \cdot \sin 50^\circ \cdot \sin 70^\circ = \frac{1}{16}$$

$$2. \sin\left(\frac{7\pi}{12}\right) \cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{7\pi}{12}\right) \sin\left(\frac{\pi}{4}\right)$$

answer

$$1. \sin 70^\circ = \cos 20^\circ$$

$$\text{So, } \sin 20^\circ \cos 20^\circ \sin 30^\circ \sin 50^\circ$$

$$\sin 20^\circ \cos 20^\circ = \frac{1}{2} \sin 40^\circ$$

$$\sin 30^\circ = \frac{1}{2}$$

$$\sin 50^\circ = \cos 40^\circ$$

$$= \frac{1}{2} \sin 40^\circ \times \frac{1}{2} \times \cos 40^\circ$$

$$= \frac{1}{4} \sin 40^\circ \cos 40^\circ$$

$$\sin 40^\circ \cos 40^\circ = \frac{1}{2} \sin 80^\circ$$

$$= \frac{1}{8} \sin 80^\circ$$

$$= \frac{1}{16}$$

$$2. \sin \frac{7\pi}{12} \cos \frac{\pi}{4} - \cos \frac{7\pi}{12} \sin \frac{\pi}{4}$$

using the identity $\sin A \cos B - \cos A \sin B = \sin(A-B)$

$$A = \frac{7\pi}{12}, B = \frac{\pi}{4}$$

$$= \sin\left(\frac{7\pi}{12} - \frac{\pi}{4}\right)$$

$$\frac{\pi}{4} = \frac{3\pi}{12}$$

$$= \sin\left(\frac{7\pi}{12} - \frac{3\pi}{12}\right)$$

$$= \sin \frac{4\pi}{12}$$

$$= \sin \frac{\pi}{3}$$

$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{2}$$

Q Prove that $2 \cos \frac{5\pi}{12} \cdot \cos \frac{\pi}{12} = \frac{1}{2}$

$$= \cos \left(\frac{5\pi}{12} + \frac{\pi}{12} \right) + \cos \left(\frac{5\pi}{12} - \frac{\pi}{12} \right)$$

$$= \cos \frac{6\pi}{12} + \cos \frac{4\pi}{12}$$

$$= \cos \frac{\pi}{2} + \cos \frac{\pi}{3}$$

$$= 0 + \frac{1}{2}$$

$$= \frac{1}{2}$$

Q $\sin \frac{7\pi}{12} \cdot \cos \frac{\pi}{4} - \cos \frac{7\pi}{12} \cdot \sin \frac{\pi}{4}$

$$\sin \left(\frac{7\pi}{12} - \frac{\pi}{4} \right)$$

$$\sin \frac{7\pi - 3\pi}{12} = \sin \frac{4\pi}{12}$$

$$= \sin \frac{\pi}{3}$$

$$= \frac{\sqrt{3}}{2}$$

$$\sin 3A = 3 \sin A - 4 \sin^3 A$$

$$\cos 3A = 4 \cos^3 A - 3 \cos A$$

$$\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A} \quad \left(\text{where } A \neq (2n+1) \frac{\pi}{6} \right)$$

T-ratios of $(A/2)$ Formulae

$$\sin A = 2 \sin \frac{A}{2} \cos \frac{A}{2} = \frac{2 \tan \frac{A}{2}}{1 + \tan^2 \frac{A}{2}}$$

$$\cos A = \cos^2 \frac{A}{2} - \sin^2 \frac{A}{2} = 2 \cos^2 \frac{A}{2} - 1 = 1 - \sin^2 \frac{A}{2} = \frac{1 - \tan^2 \frac{A}{2}}{1 + \tan^2 \frac{A}{2}}$$

$$\tan \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

$$\tan A = \frac{2 \tan \frac{A}{2}}{1 - \tan^2 \frac{A}{2}}$$

$$\tan \frac{A}{2} = \frac{\pm \sqrt{\tan^2 A + 1} - 1}{\tan A} = \tan \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

$$\cot \frac{A}{2} = \pm \sqrt{\frac{1+\cos A}{1-\cos A}}$$

Q $\sin 10^\circ \cdot \sin 30^\circ \cdot \sin 50^\circ \cdot \sin 70^\circ = \frac{1}{16}$

LHS

$$\frac{1}{2} (\sin 10^\circ \cdot \sin 50^\circ \cdot \sin 70^\circ)$$

$$= \frac{1}{2} \times \frac{1}{2} (2 \times \sin 50^\circ \times \sin 10^\circ) \sin 70^\circ$$

$$= \frac{1}{4} [\cos (50^\circ - 10^\circ) - \cos (50^\circ + 10^\circ)] \sin 70^\circ$$

$$(\because 2 \sin a \cdot \sin b = \cos (a-b) - \cos (a+b))$$

$$= \frac{1}{4} [\cos 40^\circ - \cos 60^\circ] \sin 70^\circ$$

$$= \frac{1}{4} [\cos 40^\circ - \frac{1}{2}] \sin 70^\circ$$

$$= \frac{1}{4} [\cos 40^\circ \cdot \sin 70^\circ - \frac{1}{2} \sin 70^\circ]$$

$$= \frac{1}{4} \times \frac{1}{2} [2 \cos 40^\circ \cdot \sin 70^\circ - \sin 70^\circ]$$

$$= \frac{1}{8} [\sin (40^\circ + 70^\circ) - \sin (40^\circ - 70^\circ) - \sin 70^\circ]$$

$$= \frac{1}{8} [\sin 110^\circ + \sin 30^\circ - \sin 70^\circ]$$

$$= \frac{1}{8} [\sin (180^\circ - 70^\circ) + \frac{1}{2} - \sin 70^\circ]$$

$$= \frac{1}{8} [\sin 70^\circ + \frac{1}{2} - \sin 70^\circ]$$

$$= \frac{1}{16} \quad \text{RHS}$$

Q $\frac{\sin 2\theta}{1+\cos 2\theta} = \tan \theta$ Prove that

$$\text{LHS } \frac{\sin 2\theta}{1+\cos 2\theta} = \frac{2 \sin \theta \cdot \cos \theta}{1+2 \cos^2 \theta - 1}$$

$$= \frac{2 \sin \theta \cdot \cos \theta}{1+2 \cos^2 \theta - 1}$$

$$= \frac{2 \sin \theta \cdot \cos \theta}{2 \cos^2 \theta}$$

$$= \frac{\cancel{x} \sin \theta \cdot \cos \theta}{\cancel{x} \cos^2 \theta}$$

$$= \frac{\sin \theta \cdot \cancel{\cos \theta}}{\cos^{\cancel{2}} \theta}$$

$$= \frac{\sin \theta}{\cos \theta} = \tan \theta \quad \text{RHS}$$

Q $\frac{1 + \sin 2\theta}{1 - \sin 2\theta} = \tan^2 \left(\frac{\pi}{4} + \theta \right)$

$$\begin{aligned} \text{LHS} &= \frac{1 + \frac{2 \tan \theta}{1 + \tan^2 \theta}}{1 - \frac{2 \tan \theta}{1 + \tan^2 \theta}} \\ &= \frac{1 + \tan^2 \theta + 2 \tan \theta}{1 + \tan^2 \theta - 2 \tan \theta} \\ &= \left[\frac{1 + \tan \theta}{1 - \tan \theta} \right]^2 \end{aligned}$$

$$\begin{aligned} \text{RHS} &= \tan^2 \left(\frac{\pi}{4} + \theta \right) \\ &= \left[\tan \left(\frac{\pi}{4} + \theta \right) \right]^2 \\ &= \left[\frac{\tan \frac{\pi}{4} + \tan \theta}{1 - \tan \frac{\pi}{4} \tan \theta} \right]^2 \\ &= \left[\frac{1 + \tan \theta}{1 - \tan \theta} \right]^2 \end{aligned}$$

$$\text{LHS} = \text{RHS}$$

$$9 \quad \frac{\sin 3\theta - \sin \theta}{\cos 3\theta + \cos \theta} = \tan \theta$$

$$\text{LHS} = \frac{3\sin \theta - 4\sin^3 \theta - \sin \theta}{4\cos^3 \theta - 3\cos \theta + \cos \theta}$$

$$= \frac{2\sin \theta - 4\sin^3 \theta}{4\cos^3 \theta - 2\cos \theta}$$

$$= \frac{2\sin \theta (1 - 2\sin^2 \theta)}{2\cos \theta (2\cos^2 \theta - 1)}$$

$$= \frac{\sin \theta \cos 2\theta}{\cos \theta \cos 2\theta}$$

$$= \tan \theta \quad \text{RHS}$$

HW

$$10 \quad \frac{1 + \sin 2A - \cos 2A}{1 + \sin 2A + \cos 2A} = \tan A$$

$$20 \quad \cos 6A = 32\cos^6 A - 48\cos^4 A + 18\cos^2 A - 1$$

Answer

$$1. \quad \frac{1 + \sin 2A - \cos 2A}{1 + \sin 2A + \cos 2A} = \tan A$$

$$\text{LHS} = \frac{1 + \sin 2A - \cos 2A}{1 + \sin 2A + \cos 2A}$$

Using:-

$$1 - \cos 2A = 2\sin^2 A$$

$$\sin 2A = 2\sin A \cos A$$

$$= \frac{2\sin^2 A + 2\sin A \cos A}{2\cos^2 A + 2\sin A \cos A}$$

Taking $2\sin A$ from numerator and $2\cos A$ from denominator

$$= \frac{2\sin A (\sin A + \cos A)}{2\cos A (\cos A + \sin A)}$$

canceling common terms,

$$= \frac{\sin A}{\cos A}$$

$$= \tan A$$

$$\text{LHS} = \text{RHS}$$

$$2. \cos 6A = 32 \cos^6 A - 48 \cos^4 A + 18 \cos^2 A - 1$$

$$\cos 3A = 4 \cos^3 A - 3 \cos A$$

$$\cos 6A = 2 \cos^2 3A - 1$$

Substituting the value of $\cos 3A$

$$\cos 6A = 2(4 \cos^3 A - 3 \cos A)^2 - 1$$

$$\text{Using } (a-b)^2 = a^2 - 2ab + b^2$$

$$= 2[16 \cos^6 A - 24 \cos^4 A + 9 \cos^2 A] - 1$$

multiplying by 2,

$$= 32 \cos^6 A - 48 \cos^4 A + 18 \cos^2 A - 1$$

Therefore,

$$\cos 6A = 32 \cos^6 A - 48 \cos^4 A + 18 \cos^2 A - 1$$

Inverse Trigonometry Function

Function	Domain	Range for Inverse Trigonometry Function
$\sin^{-1}$	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$\cos^{-1}$	$[-1, 1]$	$[0, \pi]$
$\operatorname{cosec}^{-1}$	$\mathbb{R} - (-1, 1)$	$[-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$
$\sec^{-1}$	$\mathbb{R} - (-1, 1)$	$[0, \pi] - \{\frac{\pi}{2}\}$
$\tan^{-1}$	$\mathbb{R}$	$(-\frac{\pi}{2}, \frac{\pi}{2})$
$\cot^{-1}$	$\mathbb{R}$	$(0, \pi)$

open bracket $()$:- endpoint not included

closed bracket $[]$:- endpoint is included

Properties of Inverse trigonometry function :-

$$\rightarrow \sin^{-1}(\sin u) = u$$

$$\rightarrow \sin(\sin^{-1} u) = u$$

Q $\sin^{-1}\left(\frac{1}{2}\right) = ?$

$$= \frac{\pi}{6} \text{ or } 30^\circ$$

another process to find the answer :-

let $\sin^{-1}\left(\frac{1}{2}\right) = u$

$$\frac{1}{2} = \sin u$$

$$u = \frac{\pi}{6}$$

CW

Q $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

let $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = u$

$$\frac{1}{\sqrt{3}} = \tan u$$

$$= \frac{\pi}{6}$$

Q $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

let $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right) = u$

$$-\frac{1}{\sqrt{3}} = \cot u$$

$$= 120^\circ \times \frac{\pi}{180^\circ}$$

$$= \frac{4\pi}{3}$$

or

$$\cot u = -\frac{1}{\sqrt{3}}$$

$$\cot(-u) = -\cot u$$

$$\cot\left(-\frac{\pi}{3}\right) = -\cot \frac{\pi}{3}$$

$$= -\frac{1}{\sqrt{3}}$$

$$= -\frac{\pi}{3}$$

$$\cot(\pi + \theta) = \tan \theta$$

$$\cot\left(\pi + \left(-\frac{\pi}{3}\right)\right)$$

$$\cot\left(\pi - \frac{\pi}{3}\right)$$

$$\cot\left(\frac{2\pi}{3}\right)$$

$$= \frac{2\pi}{3}$$

Q $\sec^{-1}(2) = ?$

$\sec^{-1}(2) = u$

$$2 = \sec u$$

$$= \frac{\pi}{3}$$

Q $\sin^{-1}(2u\sqrt{1-u^2}) = 2\sin^{-1}u, -\frac{1}{2} \leq u \leq \frac{1}{2}$

LHS :- $\sin^{-1}(2u\sqrt{1-u^2})$ — (1)

put $u = \sin \theta$ — (2)

$\theta = \sin^{-1}u$ — (3)

Now put the value of eqⁿ (2) in eqⁿ (1)

$$\sin^{-1}(2\sin \theta \sqrt{1-\sin^2 \theta})$$

$$\sin^{-1}(2\sin \theta \cos \theta)$$

$$\sin^{-1}(\sin 2\theta)$$

$$= 2\theta$$

$$= 2\sin^{-1}u \text{ RHS}$$

HW

$$Q \quad \sin^{-1}(2u\sqrt{1-u^2}) = 2\cos^{-1}u$$

Answer

$$\sin^{-1}(2u\sqrt{1-u^2}) = 2\cos^{-1}u$$

$$\text{LHS} :- \sin^{-1}(2u\sqrt{1-u^2}) \text{ --- (1)}$$

$$\text{Put } \cos \theta \text{ --- (2)}$$

$$\theta = \cos^{-1}u \text{ --- (3)}$$

Now put the value of eqⁿ (2) in eqⁿ (1)

$$= \sin^{-1}(2\cos \theta \sqrt{1-\cos^2 \theta})$$

$$= \sin^{-1}(2\cos \theta \sin \theta)$$

$$= \sin^{-1}(\sin 2\theta)$$

$$= 2\theta$$

$$= 2\cos^{-1}u \text{ RHS}$$